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TESTING OF ACCURACY OF NC MACHINE TOOLS

J. Tlusty

Lawrence Livermore Laboratory
Livermore, California

October 1980

U.S. DEPARTMENT OF COMMERCE
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TESTING OF ACCURACY OF NC MACHINE TOOLS

Jiri Tlustý
McMaster University, Ontario, Canada

October 1980

Supplement 1 of TECHNOLOGY OF MACHINE TOOLS

a survey of the state of the art
by the
Machine Tool Task Force

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TESTING OF ACCURACY OF NC MACHINE TOOLS

1.0 PAST WORK

1.1 HISTORY OF THE DEVELOPMENT OF MACHINE TOOL METROLOGY

Systematic testing of accuracy of machine tools started with Schlesinger,¹ who in 1932 established a system of measurements for acceptance testing. These were formulated from the point of view of machine tool building; they essentially checked the straightness of guideways, their mutual squareness, and their parallelism or squareness with the spindle axis. The basic measuring tools were the level and the dial gage. Even what was called straightness of motion was tested (in the vertical direction) by using a level; thus actually the variation of position of the moving body was measured instead. His book contained not only specifications of individual tests for the individual types of machine tools, but also specifications of tolerances of the individual errors. These were based on what was achievable in good machine tool building practice and what was measurable with the gages used. The most common deviation in angle was specified as 0.02 mm/m and readings with dial gages were specified with a resolution of 0.01 mm. There was an obvious need for this kind of standardization of accuracy of good machine tool building. Schlesinger's work had a tremendous impact and his book was republished with modifications several times; F. Koenigsberger took care of it after Schlesinger's death. Schlesinger's book was a basis for an extensive series of German standards for accuracy of machine tools, Russian standards, and many other national standards, and, in principle, also for the ISO standards.

A fundamentally different approach was developed in France by Salmon.² His acceptance testing concentrated not on the parts of the machine tool but on the effect machine tool motions have on the accuracy of the workpiece, and he specified tests and tolerances for finish machining of specific forms and sizes of test workpieces.

Further development made use of electronic gaging. Tlustý published a paper³ describing the use of an electronic level and of inductive pickups

for checking accuracy of spindle rotation such that it is related to the accuracy of the bore machined.

New development of metrology of machine tools started with the introduction of NC machine tools. Ericson⁴ introduced the "work zone" concept. Systematic measurements based on correct ways of defining straightnesses of motions were introduced at the Lawrence Livermore National Laboratory by Bryan and Pearson.⁵ They also included the effects of angular motions, pitch, yaw, and roll upon errors of positioning and straightness at various offsets. The concept of spindle rotation accuracy was further improved and a new apparatus for its measurement developed by Bryan, Clouser, and Holland,⁶ and by Peters and Vanherck.⁷ The whole concept was thoroughly discussed in the STC "Me" of CIRP and a unified document of CIRP was eventually published.

In various research institutions the effects of weight, clamping, and thermal deformations on accuracy were studied.⁸⁻¹⁸ The references quoted here are selected ones of the period 1965-1970. Since then, of course, there have been numerous additional papers devoted to the subject of thermal deformations. In the above period, attention was also devoted to errors generated within the control system and in the feed drive servomechanism.¹⁹⁻²¹

Because of the importance of positioning in NC machine tools and the availability of the laser interferometer, standard rules for evaluation of positioning measurements were proposed in the U.S. by the NMTBA,²² and in Germany by the VDI.²³

An extensive study attempting to summarize all the knowledge about errors in machine tool motions and about their testing was undertaken by the British Ministry of Technology under the direction of Tlusty at UMIST Manchester in 1969-1970.²⁴ The report discussed the various aspects of the general concept, magnitude of errors, spindle rotation accuracy, effects of drives, weight and clamping and thermal effects, effects of deformations under cutting force, and forced vibrations. Case histories were included and standards proposed. The concept of such tests was further developed and refined by Tlusty^{25,26} and by Mutch.²⁷

During the last decade the use of the laser interferometer for checking positioning accuracy has become a standard part of installations of NC machine tools by most leading machine tool manufacturers. General progress in acceptance testing was mainly based on new ISO specifications.

The "Me" STC of CIRP (working group in metrology) has been discussing the concepts of the three-dimensional working zone, especially for coordinate measuring machines, and a corresponding report has been published.²⁸ Papers were published introducing approaches different from the common one based on the measurements being related to the basic axes of motion of the machine tool. In Ref. 29, Hocken used measurements of positions reached in the three-dimensional zone of the machine through various coordinate combinations by measuring them from a reference fixed with the machine tool frame.

1.2 PAST RESEARCH EFFORT SIZE AND COST

Recapitulating this brief review of the history of machine tool metrology, it may be stated that a large amount of research work has been devoted to accuracy of machine tools in general and to its testing in particular. It is difficult to assess the amount of time and money spent on the latter activity. However, including the efforts of Schlesinger and of the standards bodies in Germany, France, USSR, Japan, and other countries, of the ISO, NMTBA, MTTA in Britain, and VDI in Germany, of the machine tool builders (naming just two as examples: Dixi³⁰ and Sundstrand³¹), and of research institutions (quoting a few: VUOSO in Prague, Lawrence Livermore National Laboratory (LLNL), University of Leuven, McMaster University, National Bureau of Standards, Precision Engineering Lab at Cranfield, and Technische Hochschule Darmstadt), it is obvious that something like 100 man-years may be as good a guess as any.

It is fairly correct to say that metrology and testing of machine tools has reached a level which is satisfactory for most purposes. It might now be possible to formulate a system of testing which could be almost universally acceptable and which could last for, maybe, another decade. An indication of what it might look like is given in Section 2.0, which deals with the state of the art. However, further progress is inevitable; it will probably concentrate on more automation of testing, more statistics, and still better measuring instruments.

2.0 STATE OF THE ART OF ACCURACY TESTING OF MACHINE TOOLS-- PROPOSAL OF A STANDARD TEST

2.1 APPROACH

To summarize the state of the art of accuracy testing, a concept, basic explanations, definitions, methods, and a list of tests are presented here as a general proposal for a standard test that corresponds to the present state of the art of metrology of numerically controlled machine tools. The author will present essentially an approach and formulations for a system based on his previous publications resulting from his theoretical work and practical testing experience at UMIST in Manchester and at McMaster University, and which has developed in his close contact with R. McClure and J. Bryan in the impressive research and metrology development work at LLNL. The author also had discussions of the various aspects of the system in the "Me" STC of CIRP. To the best knowledge of the author there are no contradictions between this system and the metrological practice of LLNL or of WZL Aachen.

2.2 THE BASIC CONCEPTS OF THE TEST SYSTEM

The three main basic features of this system are:

(a) It is a non-machining system of tests which could be called "tracing of a universal master part."

(b) It is an essentially non-statistical test which assumes that the individual sources of errors and their motions can be, to a reasonable extent, isolated and systematically combined.

(c) The tests are formulated from the user's and not from the builder's point of view, and they are related to the accuracy of the workpiece.

Let us explain these features.

In system feature (a) an alternative would be a test in which one or more test workpieces are machined and subsequently measured. There are several difficulties associated with such an approach. The accuracy of the workpiece is affected by tool wear, thermal deformations of the machine due to both its internal sources and to the heat generated by the cutting process and transmitted through chips and coolant, thermal distortions of the workpiece itself, deformations of the machine tool caused by the weight of the workpiece,

deformations of the workpiece caused by its clamping, and deformations caused by cutting forces. These effects depend very much on the choice of cutting conditions as well as on the size and shape of the workpiece. It is therefore very difficult to assess what could optimally be achieved.

There are further problems:

- The chosen workpiece can represent to only a limited extent the size and shape of all the parts to be made on the given machine.
- It is difficult to measure the workpiece. Metrology of parts is different from metrology of machine tool motions and essentially it is necessary to provide a coordinate measuring machine which simulates a non-machining machine tool to measure the workpiece. The expense of the measurement of the part is additional to the expense of manufacturing the test piece.
- The non-machining accuracy test excludes effects like tool wear, heat of the machining process, and part clamping and it can be arranged so as to determine thermal deformations of the machine tool; it is based on the most universal workpiece and is rather inexpensive.

Therefore, machining and measuring of a test part should be limited to testing of special-purpose machines where the technology of the machining operation is well defined and only size and shape of workpiece is involved.

In feature (b), the main question is how far can the individual effects be separated and subsequently superimposed and, mainly, how many of the existing effects have been satisfactorily determined and how many of them remain "statistical."

Most effects are separately associated with the individual coordinate motions and tests can conveniently be made along each of the individual coordinates.

Let us briefly discuss these points. They will be analyzed in more detail later. The positioning error contains the following components: the cumulative error, the periodic error, the dead zone, and scatter (repeatability). The first three of these are determined by the positional feedback transducer and its kinematics and by the dynamics of the drive, all of which are rather systematic. The dead zone may also be affected by the play in guideways as combined with yaw, and this again is predominantly systematic. Scatter is treated statistically and it usually is negligible. Error of straightness of motion depends mainly on the shape of the guideways,

which except for a reservation mentioned later is a systematic influence, if weight deformations caused by the workpiece are separated. Errors of parallelism and squareness depend mainly on the structure of the machine and as such are also systematic. Pitch, yaw, and roll can be considered in a similar way. These may also be affected by the flexibility of the moving bodies, which again is a systematic effect.

In system feature (c), from the user's point of view, the whole test has to be arranged so as to consider the errors of the workpiece resulting from the errors of the machine tool motions. This is obtained by arranging the test as that of master part tracing, which is explained in Sec. 2.3. To illustrate the distinction from a builder's point of view, let us mention the instances whereby instead of checking the straightness of guideways we check straightness of motions, and instead of checking positioning at the location of the feedback transducer we check the distance between spindle and workpiece, etc.

However, even though the user's point of view is primary, it is often possible and useful to correlate individual "workpiece" error components with the errors of individual machine tool elements, e.g., a periodic positioning error corresponding to the axial run-out of the thrust bearing of the leadscrew. Another "by-product" of systematically identifying repeatable errors is that software compensation can be ultimately prepared.

2.3 THE MASTER PART TRACING APPROACH

Master Part Tracing

In Ref. 32 Bryan describes a technique for checking the errors of a copy system; it is Master Part Tracing and his Master Part is a circular template used to simulate a semicircular workpiece. It is possible to generalize this concept and arrange all our testing so that we will simulate machining by replacing the tool by a gage and the workpiece by an ideally accurate Master Part (or at least one with precisely known inaccuracy). In this way we remove all the effects of the cutting process while retaining all the motion errors inherent in the machine tool. If during the motion of the gage simulating the machining motion over the Master Part the reading of the gage did not vary at all, then the motion would be ideally accurate. Any variation of the gage

reading constitutes an error. This technique also implies that all measurements are taken relative between the tool (the gage) and the workpiece (the Master Part).

For the important question of how to create a universal Master Part the answer is: to construct it at various locations with master scales (laser interferometer beams) and straightedges. We will see in the following how such locations are chosen.

Reference Surfaces, Reference Motions

The next question is how to align the Master Part, or rather its elements (the straightedges and laser beams) with respect to the machine tool structure. As the Master Part represents the workpiece it can be aligned with respect to that part of the machine onto which the workpiece is clamped: the table, the floor-plate, the chuck fixed to spindle end. These parts may serve as reference surfaces. If they do serve in this quality they have to possess a certain accuracy. For example, the table of the machine in Fig. 1 must have a certain flatness and its T-slots (or at least one of them, the reference one) must be straight within a tolerance. It is obvious that if the accuracy of a machine tool is related to such a reference surface it cannot be specified better than to the tolerance of the reference surface. Moreover, the accuracy of the table in Fig. 2 cannot be uniquely specified unless the table is rigid. Otherwise, its flatness varies during the X motion, and maybe even during the Y motion.

It is possible not to use a reference surface but to assume an absolutely rigid workpiece supported on a flexible table so as to not be affected by its deformations, e.g., by resting on three selected points (or an approximation to this). In such a case the Master Part would be constructed on a rigid fixture supported on the same three points and aligned with respect to the motions in the individual coordinates instead of to a reference surface. It is explained later how this is done.

These distinctions have to be a part of the specification of the machine tool.

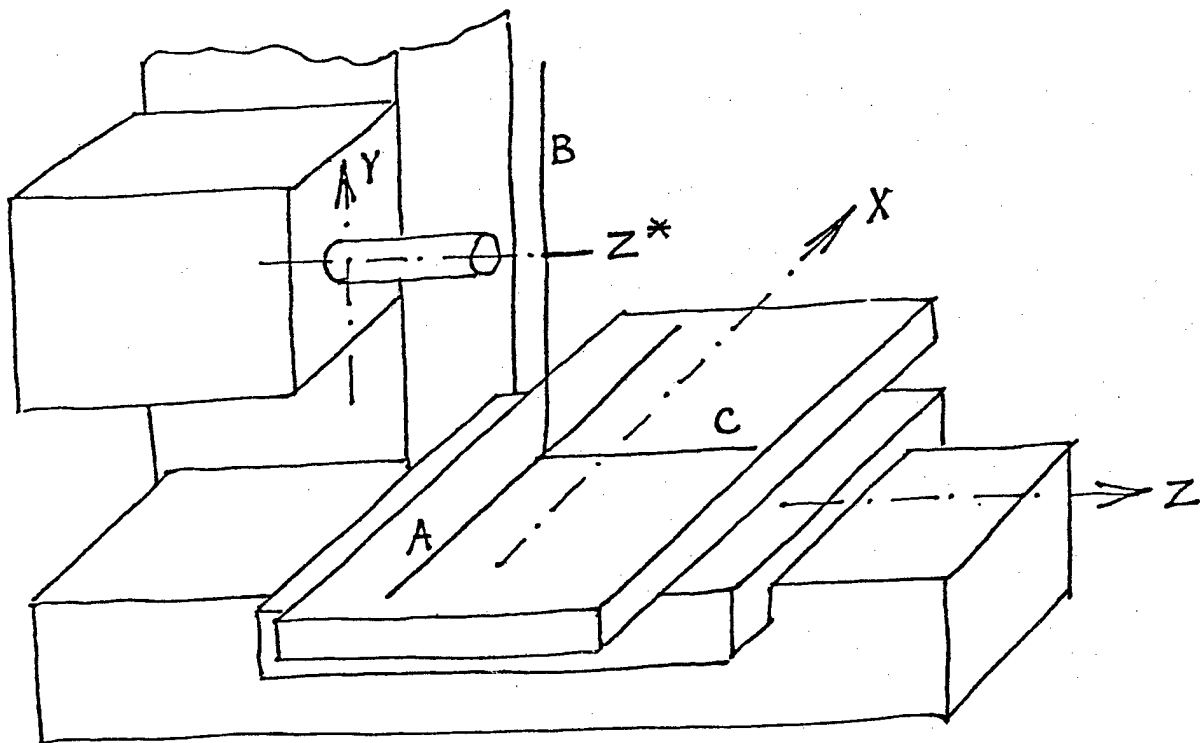


FIG. 1. Horizontal boring mill: axis and motion nomenclature.

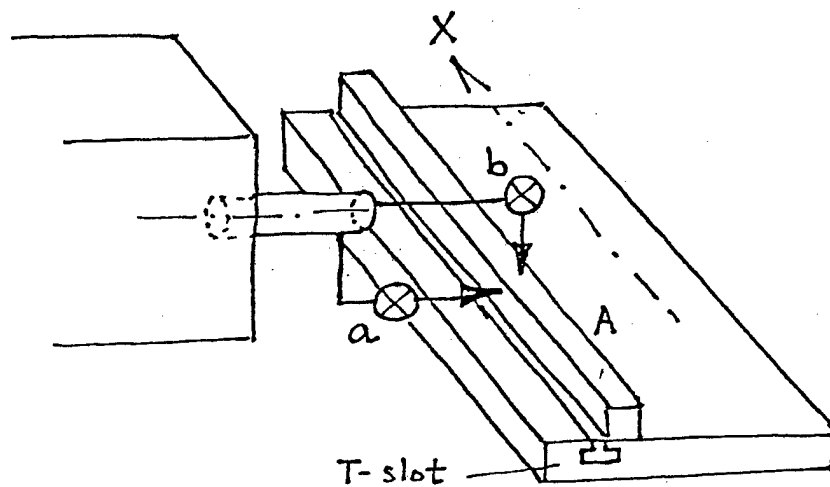


FIG. 2. Slide-straightness measuring arrangement.

3.0 BASIC MEASUREMENTS: TRANSLATIVE ERRORS

There are three basic measurements associated with every coordinate motion and they are carried out along selected lines. Let us choose the X coordinate motion of the horizontal boring machine depicted in Fig. 1. It is the motion of the table and we shall first assume that the table is a rigid body. A line A, generally parallel with X motion, is first selected; it represents a line of the Master Part at coordinates z_A and y_A (measured from some selected origin of coordinates). Any point along this line will, in addition, have a nominal coordinate x.

3.1 MEASUREMENTS OF STRAIGHTNESS OF MOTIONS

Let us materialize line A by a straightedge with ideally straight sides both at the top and towards the headstock, and move in X with gages a and b sliding on the straightedge (Fig. 2). The readings of these gages will constitute what are generally called measurements of "straightness of motion X in the Y and Z directions." Any variations in these readings will represent deviations from ideal locations of the points on a line as it would be machined on a workpiece instead of the ideal line A.

Let us call these deviations δ , with a subscript indicating their direction and parentheses enclosing the coordinate of the motion as well as the denotation of the line or of its coordinates; the latter data may be omitted from the notation and given separately. The alternatives of notations are:

$$\delta_y(x), \text{ on line A, (preferable); } \delta_y(x,A); \delta_y(x,y_A,z_A) \quad (1)$$

$$\delta_z(x), \text{ on line A, (preferable); } \delta_z(x,A); \delta_z(x,y_A,z_A) \quad (2)$$

Let us, first, suppose that the table is used as a reference surface, that it is ideally flat and has an ideally straight T-slot, and that the straightedge is aligned exactly parallel with the surface of the table and with the T-slot.

Let us carry out the $\delta_y(x)$, $\delta_z(x)$ measurements during one direction of the X motion. The record of the gage signal will look like that in Fig. 3 with the gage zeroed at the beginning of the +X motion.

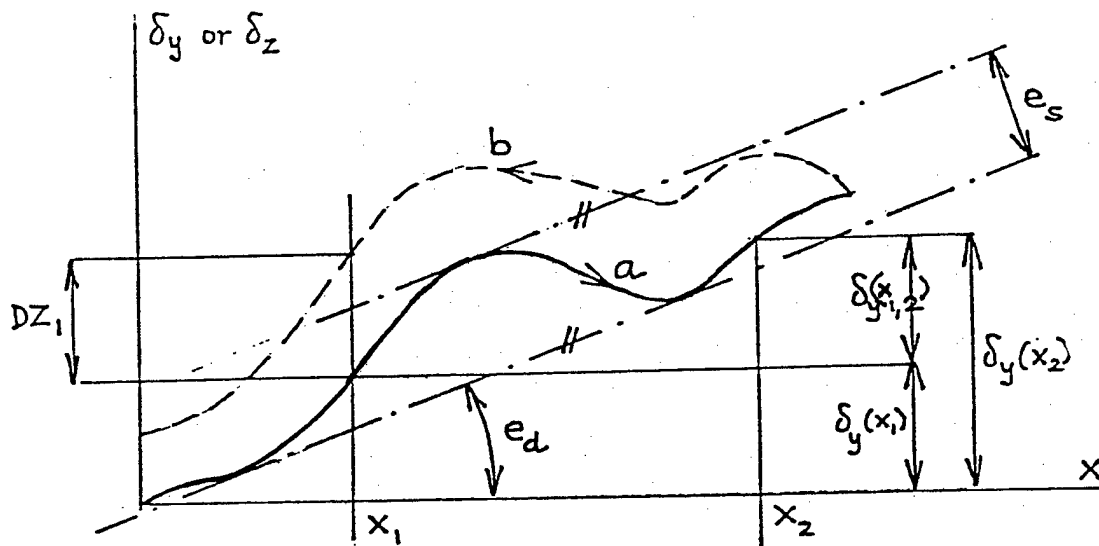


FIG. 3. Simulated straightness recording $\delta_z(x)$ or $\delta_y(x)$.

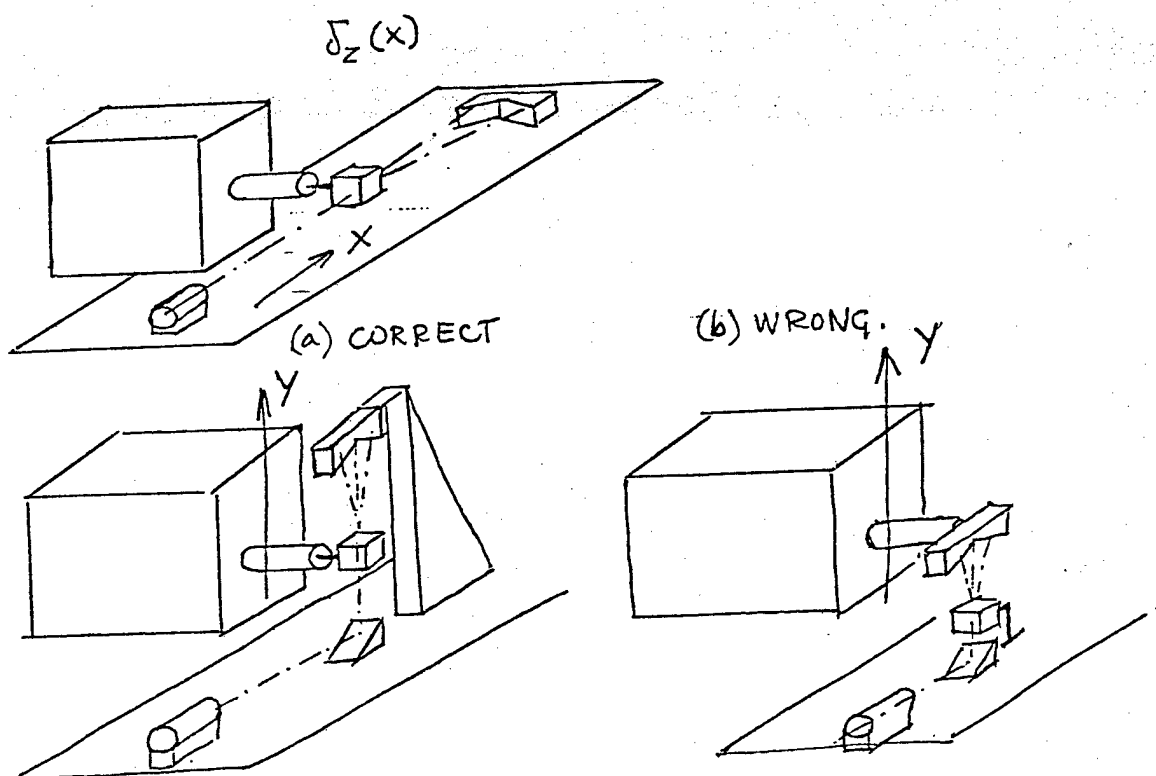


FIG. 4. Straightness interferometer arrangements.

Let us look at the curve a alone which corresponds to the +X direction of motion. From the machine tool builder's point of view it can be considered as consisting of a "straightness of motion error" e_s (the minimum distance of two parallel lines enclosing the curve) and an "error of direction" e_d^* (the slope of these lines). In the case of $\delta_y(x)$, e_d would be the error of parallelism of the table surface with motion X; for $\delta_z(x)$, it would be the error of parallelism of the T-slot with motion X.

However, from the user's point of view, i.e., from the point of view of the accuracy of the workpiece, these distinctions have no sense at all and the functions $\delta_y(x)$ or $\delta_z(x)$ have to be simply considered as errors of location of points of the workpiece. Later we will discuss the way to evaluate them.

Still, it is useful to realize right now that the difference [$\delta_y(x_1) - \delta_y(x_2)$] between the errors of points 1 and 2 located at coordinates (x_1, y_A) and (x_2, y_A) is the y-component error of the distance of the two points (which in this case ideally should be zero).

Confusion might arise in nomenclature. The measurement itself is traditionally called "measurement of straightness of motion," while in its interpretation only the part e_s is usually called "error of straightness." In order to avoid confusion while not at all attempting to finalize a new name we will, in this paper, call the errors perpendicular to the line of measurement--like here the $\delta_y(x)$ and $\delta_z(x)$ --lateral[†] errors, while keeping the traditional name of measurement of straightness for the test itself.

If, instead of a straightedge, a straightness of motion measuring mode of a laser interferometer is used, it is absolutely necessary to correctly locate its individual parts. As shown in Fig. 4(a) the axis of symmetry of the reflector replaces the straightedge (a line on the workpiece) while the straightness interferometer beam splitter represents the gage (the tool). Thus, e.g., on a bed-type milling machine, for the measurement of the table

*In the notations e_s and e_d the letter e is used as distinct of the Greek letters δ and ϵ for translative and angular errors; the former represents simple numbers and the latter represents functions of a (moving) coordinate.

[†]The name lateral is due to Hocken who, however, uses it in a slightly different sense.

motion X the reflector is attached to the moving table and the interferometer to the spindle. For the Y motion, again the reflector has to be attached to the table, via e.g., an angle plate. It would be very wrong to arrange as shown in Fig. 4(b).

Similarly to the measurements of straightness of motion X, the measurements of straightness of motions Y and Z will be carried out with the gage attached to the spindle and tracing straightedges located on the table along selected lines B and C in the directions Y and Z (see Fig. 1). The positions of these straightedges, first of all, have to be square to the position used for the measurement during motion X (because they represent lines on the Master Part at a specific point in orthogonal directions). Thus, e.g., the one for motion Z will be parallel with the table surface and square to the T-slot, and the one for motion Y will be square to both the table surface and the T-slot.

In the case where the table surface is not used as a reference surface it is possible to choose one of the motions as primary. Let us choose the X motion. Then the straightedge will be supported on the table so as not to be affected by its deformations and it will be aligned for minimum errors $\delta_y(x)$, $\delta_z(x)$, i.e., so that e_d (see Fig. 3) is zero and one of the boundary lines e_s lies on the X axis. Let us then choose motion Y as secondary. The corresponding position of the straightedge will be square to that during the X motion and it will also make the slope e_d of the error $\delta_z(y)$ zero. The position of the straightedge for motion Z will then be square to the two previous ones.

If it is agreed that the table is not rigid, then it cannot serve as reference. The Master Part and its components, e.g., the straightedges, have to be rigid enough and so supported on the table as to not be influenced by its deformations. The selection of the supports, which should also apply to all workpieces, must be a part of the specification of the machine. If measurement is done using the "straightness laser interferometer," its reflector (and most suitably also the laser) has to be attached to a rigid beam supported on the selected points of the table.

3.2 POSITIONING MEASUREMENTS

Referring to Fig. 1, the deviations in the direction X of points on the line A represent an error of distance travelled by motion X from a chosen

origin. These deviations are nowadays generally measured using the laser interferometer. The corner cube reflector is attached to the spindle in place of the tool and the remote interferometer (the laser beam) represents the line A on the workpiece.

A record of such a measurement can look as shown in Fig. 5 where measuring during the motion X in one direction only is represented. It is shown that, e.g., the difference $\delta_x(x_1) - \delta_x(x_2)$ represents the error of distance between points x_1 and x_2 .

3.3 ESTABLISHING REFERENCE DIRECTIONS:

MUTUAL SQUARENESS OF MOTIONS

The concept of mutual squareness or parallelism of coordinate motions is, in essence, one of the builder's point of view and it would not, in principle, be necessary at all for the user's (workpiece accuracy) point of view. However, in this latter case it is necessary to establish the reference directions relating to the Master Part.

In reference to Fig. 1 we speak about lines A, B, and C as mutually perpendicular elements of the "Master Part" passing through a point. These lines are the reference lines for the measurements of straightness of motion (defined via lateral errors). They serve also as references for position measurements. If there is a small deviation in the direction of the laser beam from the ideal reference lines a cosine error of second order results which is mostly negligible.

Whether line A is initially established with respect to the T-slot in the table or with the X-motion, it must be perpendicular to line B, and finally with the one remaining perpendicular axis in the Y-motion direction. It is necessary for A, B, and C to be mutually perpendicular and the aligning of these axes is a problem.

Several procedures have been developed to accomplish the task, although more ingenious techniques may be developed in the future. We will describe here a technique using a precision square as an aid. Another useful technique used at LLNL is based on a precision rotary table which carries the straightedge and indexes by 90° . The technique described here is valid whether the straightness measurement is done using a straightedge or using the laser interferometer.

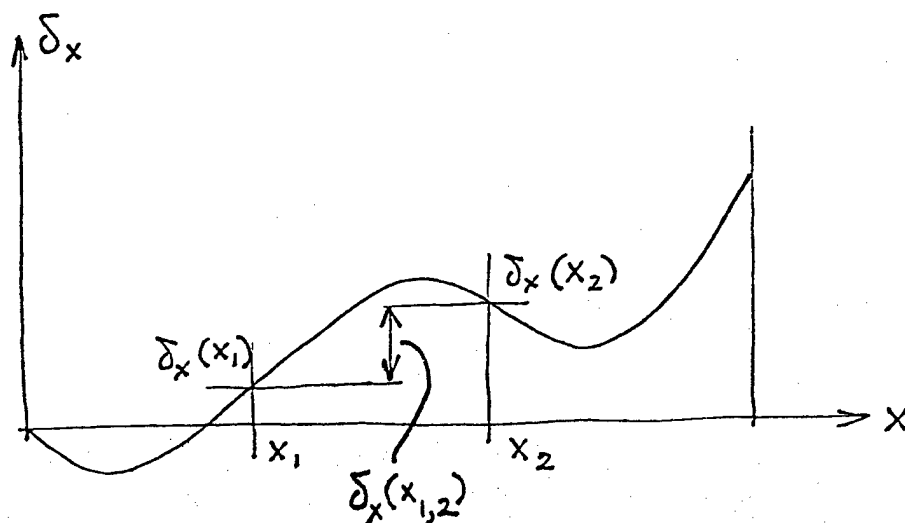


FIG. 5. Simulated position-measurement recording.

The procedure illustrated here is for the case of lines A and C in the X,Z plane shown in Fig. 6(a) and it applies in an analogous way to all the other squarenesses. First, measurements $\delta_z(x)$ and $\delta_x(z)$ are done along lines A and C while the corresponding straightedges (or laser beams) are aligned with their references using various physical means as well as it is practically possible. It is assumed here that these measurements are carried out along the whole travel lengths while the precision square to be used in the second step is, usually, of a smaller size. Let us assume that the record of the two measurements $\delta_z(x,A)$ and $\delta_x(z,C)$ look as given in Fig. 6(b) and (c). For each of them the two parallel lines enclosing the record of error of straightness are drawn and angles β_1 and β_2 are noted of their inclination to present axes of measurement. The directions of deviations δ_z, δ_x are established as in Fig. 6(a) in such a way that a signal of a gage attached to the spindle is read as positive if it represents a deviation of the straightedge (of the Master Part) in the positive direction of the corresponding coordinates. The dashed field represents the location of the precision square on lines A and C, between points 1, 2 and 3, 4 respectively ($2 \equiv 3$).

In the second stage the two measurements are repeated using the precision square. The results of these measurements are entered in the two graphs as $\delta_z^*(x)$ and $\delta_x^*(z)$. In the case in which the table T-slot is used as reference the square is aligned on line A parallel with the T-slot.

Let us note the values of β_1 and β_2 and those of the angles γ_1 and γ_2 . The latter two represent angular differences of records obtained from the δ straightedge lines (laser beams), and the former two from the square represented by δ^* lines.

(a) In the case of physically aligning A with motion X we must make $\beta_1 = 0$ by rotating the straightedge so that the record $\delta_z(x,A)$ moves toward the axis x. This will increase the misalignment between the measurements with the straightedge and the square to $(\gamma_1 + \beta_1)$. For the purpose of evaluating the graph $\delta_z(x,A)$ an equivalent rotation of coordinate axes from x to x' is possible and the error graph remains unchanged.

For proper reference of the straightedges to the square it is now necessary to adjust the difference between the measurements $\delta_x(z,C)$ and $\delta_x^*(z)$ through γ_2 and angle $(\gamma_1 + \beta_1)$ where points 2 and 3 coincide.

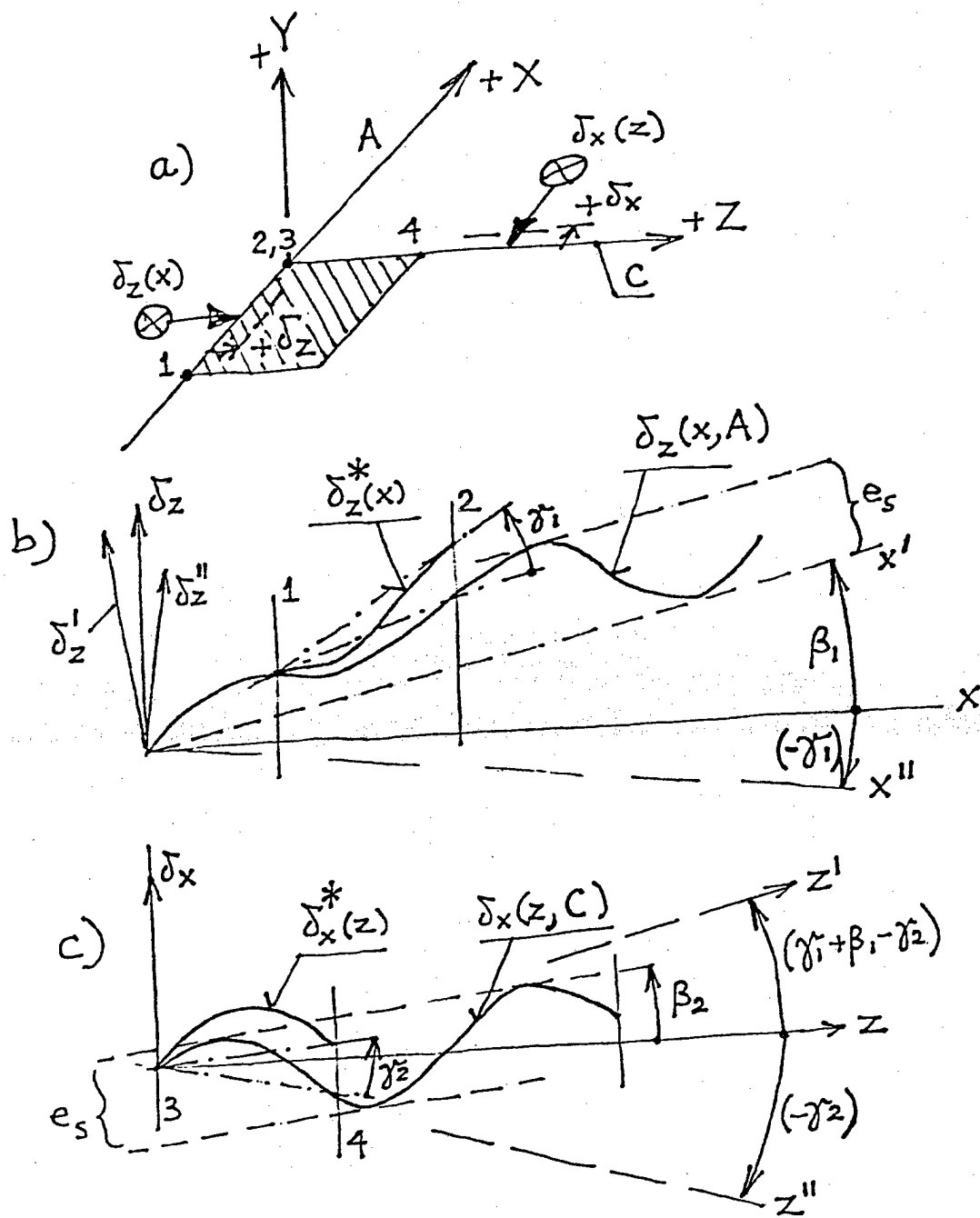


FIG. 6. Simulated mutual-squareness-of-motion recording.

This means to rotate the graph of $\delta_x(z, C)$ by $(\gamma_2 - \gamma_1 - \beta_1)$, or else to rotate the coordinate axes by $(\gamma_1 + \beta_1 - \gamma_2)$ to z' without moving the graph itself.

It is now also possible to evaluate the error of squareness of the motions X and Z as

$$e_{sq}(X, Z) = \gamma_1 + \beta_1 - \gamma_2 - \beta_2.$$

(b) In the case of aligning A with the table T-slot and having aligned the precision square with this slot, then obviously the graph of $\delta_z(x, A)$ must be moved so as to coincide with $\delta_z^*(x)$ which was measured against the true reference. This corresponds to moving axes x, δ_z to x'', δ_z'' by rotating them by $(-\gamma_1)$.

Consequently, also the graph $\delta_x(z)$ must be rotated so as to identify $\delta_x(z, C)$ with $\delta_x^*(z)$. This corresponds to rotating the coordinate axes of the graph to z'' by $(-\gamma_2)$.

The error of squareness of motions X and Z is again

$$e_{sq}(X, Z) = \gamma_1 + \beta_1 - \gamma_2 - \beta_2.$$

3.4 LATERAL ERRORS

If the measurement of an out-of-line error as it is shown in Fig. 7 is measured also during the same coordinate motion in the reverse direction, usually a line (2-3) different from the first measurement (1-2) is obtained. If the (+) motion is repeated, line (3-4) may be obtained. If, subsequently, motion in the (-) direction is carried out over a shorter distance (4-5) and then in (+) over (5-6) lines as shown may be obtained. The span between lines $\delta_z(+x)$ and $\delta_z(-x)$ could be called the Dead Zone and it may be caused by a play in the guideways and a yaw motion due to a moment between the feed driving force and friction in guideways. Repeated measurements do not yield exactly repeatable results. However, the scatter is usually very small. The area between the extreme lines is the error field.

Taking into account the nature of the origin of the out-of-line errors and the available techniques of their measurement the following is formulated as Rule 1:

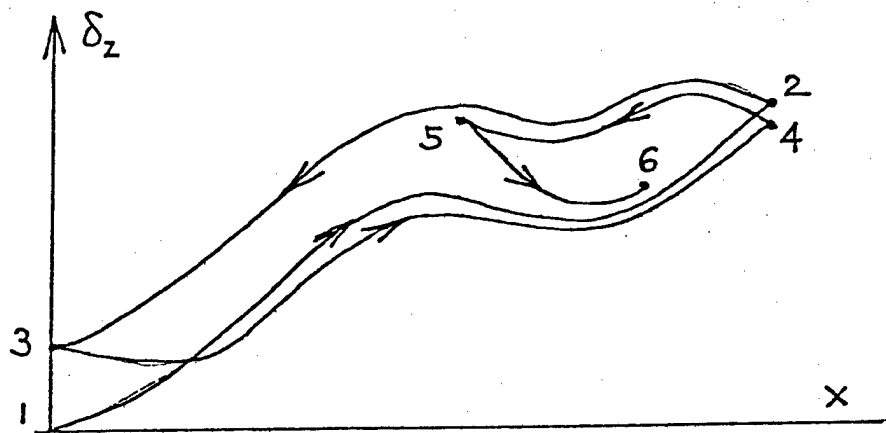


FIG. 7. Out-of-line-error simulation.

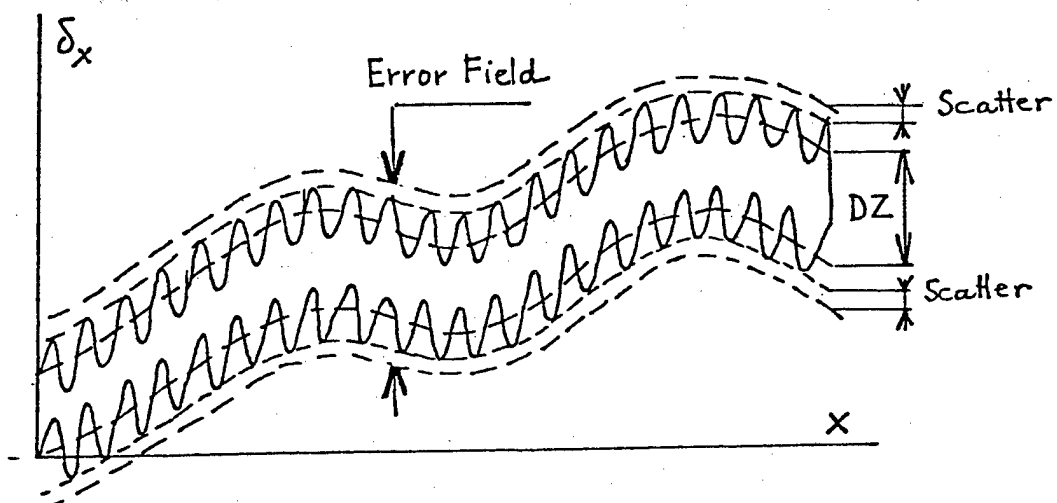


FIG. 8. Positioning-error recording, showing scatter and dead zone (DZ).

A basic straightness measurement (measurement of a lateral error) is made once in each direction of motion. The motion is continuous, its speed corresponds to a rather high feed rate (relative to the use of the particular machine) and a continuous record is made. For the recording preferably a gage with an electric output signal is used, which is recorded using a chart recorder with reversible paper motion.

3.5 POSITIONING ERRORS

The positioning error usually contains periodic components, a dead zone and some scatter (see Fig. 8). The periodic components are derived from typical periods in the feedback transducer and its kinematics. Typical examples are: for feedback derived from leadscrew rotation, once per leadscrew revolution (axial run out of the thrust bearing, one revolution of synchro or its second harmonic); for feedback derived from measuring rack, once per pinion revolution (a bent shaft of the pinion), etc. The dead zone is due to flexibility of drive and friction in guideways. Scatter is due to variability of friction and variability of the dynamics of the servodrive. It is usually small.

Positioning is usually measured in steps of motion. The corresponding recommendation is expressed as Rule 2:

1. Measure once in each direction of motion in steps corresponding exactly to the largest period of the periodic error (to be determined from the kinematics of the feedback).
2. Select two locations along the travel and measure into each of them once in each direction of motion over a travel described in step 1, 20 times.
3. If there is another much shorter important period, measure it also using that period over 20 steps.
4. Select two locations along the travel. They can be the same as in step 2. Position repeatedly to the same position alternatively from both directions, 10 times in each (see Fig. 9 for a description of the technique). Evaluate dead zone and scatter as 3 σ .
5. Combine results of all above measurements superimposing 1,2,3, and 4 into an error field.

Measurement 1 is to be made using rapid traverse, measurements 2 to 4 using medium feedrate.

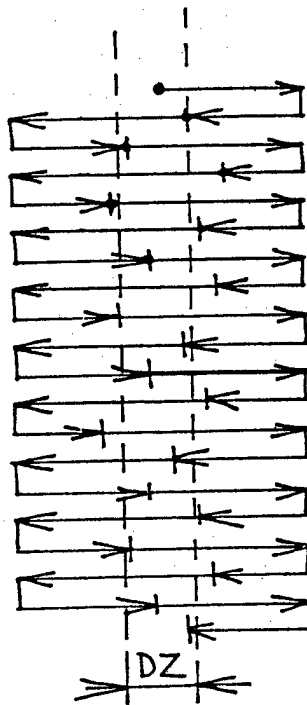


FIG. 9. Dead zone and scatter determination technique. In two points, located at 1/4 and 3/4 of the travel, position 20 times into each of these points (10 times in the positive and 10 times in the negative direction). Use feedrate 150 mm/min. In between, move ± 5 mm away.

In each point evaluate dead zone DZ_i as the difference between the average positions reached in the two directions. Take the larger of DZ_1 and DZ_2 as DZ .

All the 20 readings of the test are grouped as ten readings δ_i , $i = 1$ to 10, in each of the four groups $j = 1$ to 4 (the plus and minus positioning in the two selected points). In each group there is an average δ_{avj} . Evaluate scatter as

$$S = 3\sigma = 3 \sqrt{\frac{\sum (\delta_{ij} - \delta_{avj})^2}{20}}$$

where the summation is carried out for differences between each reading and the corresponding average.

3.6 TOLERANCE REQUIREMENTS AND EVALUATION OF TRANSLATIVE ERRORS ALONG A LINE

There are two fundamental approaches to error evaluation:

(a) A comparison of the error with a specified tolerance is made and a statement derived whether or not the tolerance is specified.

(b) The error is defined as measured.

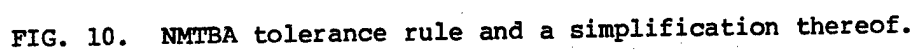
Approach (a) is analogous to the most common way of evaluating workpiece accuracy. Approach (b) is useful if a correction of the error is intended.

Here, first of all, the tolerance approach is presented.

Let us start with a brief discussion of the NMTBA tolerance rule for positioning as it is reproduced in Fig. 10a. There, the positioning error field (including scatter) is plotted in coordinates x (length of motion in X) and δ_x (error of x). The rule requires that a double sided template T , if shifted along the error figure, should always be able to include the whole of it. The tolerance template is shaped to allow for constant error Δf_x along a length b and for an additional error increasing in proportion to the distance extending beyond b . The template is free to be shifted in the directions of both coordinate axes of the figure while remaining parallel to itself. This is significant in that distance x and error δ_x are taken as relative only between any two points. In Fig. 10b a simplification of this rule is suggested in the form of a template where distance b is diminished to zero. This is comparable to the ISO system of tolerances of dimensions. Figure 10c expresses graphically the IT 5 class tolerance δ as a function of the dimension D (curve a). This function is nonlinear. However, it has two fundamental features: the tolerance does not decrease to zero when dimension D approaches zero; the tolerance increases with the dimension. Line b represents a linearization of the curve a. The tolerance template of the form given in Fig. 10b is the interpretation of the rule expressed by line a in Fig. 10c if both the tolerance and the dimension are taken in the relative way. Let us accept it for the following generalization to a three-dimensional working zone.

The mathematical expression of the template form depicted in Fig. 10b is:

$$|\delta_x(x_1) - \delta_x(x_2)| < A_{xx} + K_{xx}|x_1 - x_2| \quad (3)$$



where x_1 and x_2 are the coordinates of any two points along a positioning path in the direction X and $\delta_x(x)$ is the error in the direction X of the positions x. Equation (3) can be read: the absolute value of the error of the distance of two points (whether the distance is greater or smaller is not relevant) may be equal to or greater than a constant value A_{xx} plus another value proportional by K_{xx} to the distance (while it is irrelevant in which direction the distance is taken as positive).

The lateral errors resulting from the straightness measurement have, from the point of view of workpiece accuracy, exactly the same significance as the positioning error. Furthermore, the character of the out-of-line error field is similar to that of the positioning error field. Therefore, the NMTBA tolerance rule may be extended using the notation A_{yx} , A_{zx} and K_{yx} , K_{zx} for the lateral constants:

$$\begin{aligned} |\delta_y(x_1) - \delta_y(x_2)| &< A_{yx} + K_{yx} |x_1 - x_2| \\ |\delta_z(x_1) - \delta_z(x_2)| &< A_{zx} + K_{zx} |x_1 - x_2| \end{aligned} \quad (4)$$

These rules may also be graphically expressed by a "bow tie" template. The interpretation of the rules is such that (see Fig. 11) when moving in the X direction from point x_1 to point x_2 we end up within a box symmetrically located around the ideal location of x_2 and its sides a, b, c are obtained by Eqs. (3) and (4), respectively.

$$\begin{aligned} a &= A_{xx} + 2K_{xx} |x_1 - x_2| \\ b &= A_{yx} + 2K_{yx} |x_1 - x_2| \\ c &= A_{zx} + 2K_{zx} |x_1 - x_2| \end{aligned} \quad (5)$$

all of them having terms proportional to the distance of points x_1, x_2 .

The values of A and K constants have to be given in the specification of the machine. For a recommendation of these values see Sec. 9.0.

The evaluation method presented in the preceding text is recommended for a general assessment of machine tool accuracy. However, for the purpose of modifying the measured errors, i.e., assuming the builder's point of view or

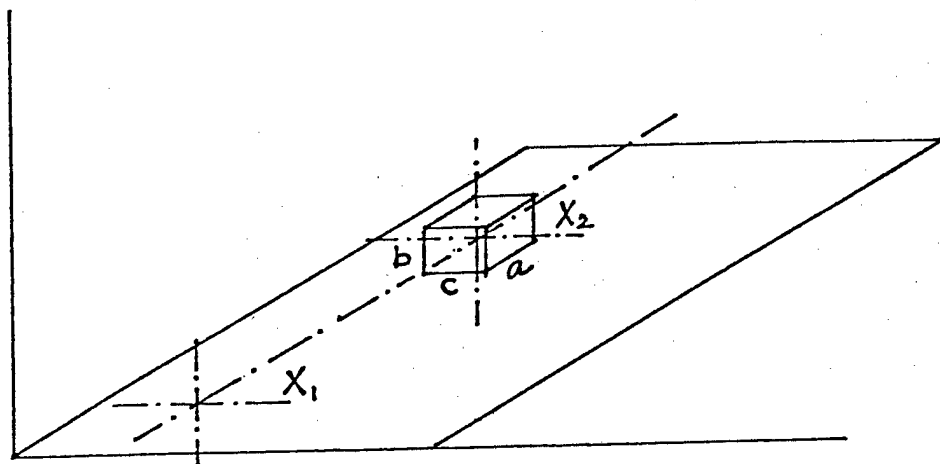


FIG. 11. NMTBA tolerance rule applied to a 3-D space.

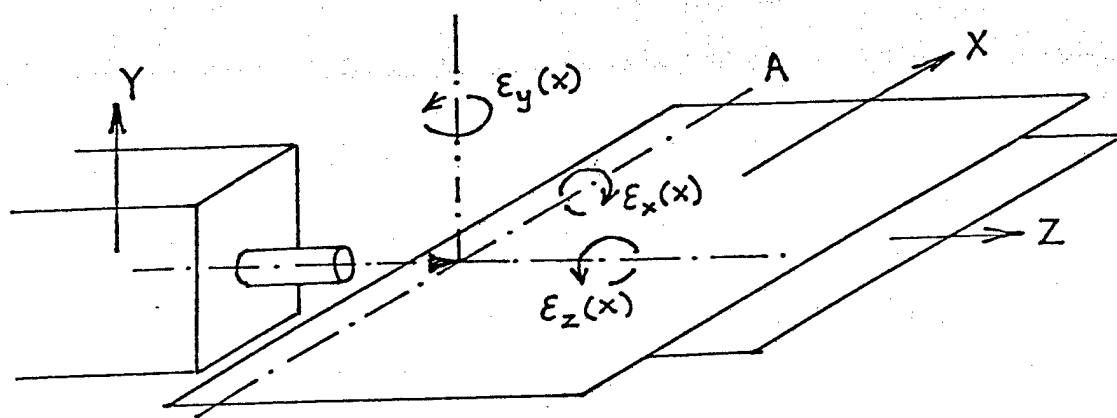


FIG. 12. Horizontal boring mill: axis and rotational motion nomenclature.

for a CNC software compensation of the errors the individual components of the errors can be determined in various ways as:

- Lateral errors. For each direction separately, graph: the errors; or the components of error of straightness e_s and error of direction e_d ; the dead zone. For error of straightness, determine its maximum rate $\Delta e_s / \Delta x$.
- Positioning errors. For each direction separately, graph: the errors; or the cumulative (coarse step error), its maximum point to point (PTP) value and its maximum rate $\Delta \delta_x(x) / \Delta x$; the dead zone; the scatter.

4.0 THE ANGULAR (ROTATIONAL) ERRORS; THEIR EFFECT ON TRANSLATIVE ERRORS THROUGHOUT THE WORKING SPACE

The motion of the table is not exactly translative and straight as it ideally should be. If the table is considered rigid then the errors of its motion X may be characterized by the three translative errors along line A mentioned in the preceding Sec. 2.3, and, in addition, by three angular motions, see Fig. 12. Let us denote them by the letter ϵ and a subscript denoting the axis of rotation and the axis of motion in brackets. They are:

$\epsilon_x(x)$, the roll
 $\epsilon_y(x)$, the yaw
 $\epsilon_z(x)$, the pitch

Each of them is taken positive clockwise when seen in the positive direction of its axis of rotation. The angles of rotation vary with the coordinate X of the table motion and the axes of rotation pass through the "tool" point. Because of these errors, associated with the motion X measurements of the translative errors $\delta_y(x)$, $\delta_z(x)$, $\delta_x(x)$ along other lines than line A will be different depending on the offsets. When these types of effects (motion), caused by angular errors that occur during travel along a particular coordinate, affect translative errors measured during the same coordinate motion they are called primary effects.

The angular errors associated with a particular coordinate motion will also affect some translative errors measured during another coordinate motion. These effects will be called secondary.

4.1 PRIMARY EFFECTS

An exact formulation of which angular error motions affect which translative motions due to which offsets is given in the Appendix. Here an illustration is given for the case of a horizontal boring machine with three coordinate motions X (table, horizontally), Y (headstock, vertically), Z (saddle, horizontally) and a setting motion Z^* (spindle extension), see Fig. 13. Let us assume that translative errors were first measured along lines A_1 (motion X), B_1 (motion Y), C_1 (motion Z). As mentioned before, the angular errors affect translative errors through offsets of lines along which the translative measurements are made. An offset in this relationship is defined as a difference of the location of the tool with respect to the corresponding carrier of the motion: table, saddle, headstock.

To explain the type of effect, let us take the positioning measurements $\delta_x(x)$ along lines A_1 and A_2 which are offset in direction Y by $b = 1$ m, see Fig. 14(a). Let us assume the positioning error measured along A_1 as given in graph b) and the pitch error as given in graph c). The positioning error along line A_2 will then be as given in graph d) because

$$\delta_x(x)_2 = \delta_x(x)_1 + b \cdot \epsilon_z(x). \quad (6)$$

If in Eq. (6) the maximum possible offset b_{\max} is used which is equal to the Y travel, the difference between $\delta_x(x)_1$ and $\delta_x(x)_2$ is maximum. The difference being due to an angular motion the error $\delta_x(x)$ along any line between A_1 and A_2 is a linear interpolation of $\delta_x(x)_1$ and $\delta_x(x)_2$:

$$\delta_x(x) = \delta_x(x)_1 + \frac{\delta_x(x)_2 - \delta_x(x)_1}{b_{\max}} \cdot b \quad (7)$$

where b is the offset between $\delta_x(x)$ and $\delta_x(x)_1$.

Correspondingly an important statement can be made, Rule 3:

If two translative error measurements along lines maximally offset are both within a specified tolerance a measurement of the same kind of error along any line between the two extreme ones is also within that same tolerance.

Let us now examine the extreme offsets in the horizontal boring machine. The offsets and angular errors in Table 1 combine to affect the translative

TABLE 1. Combinations of angular errors and offsets that have primary effects on translative errors.

During motion X (Fig. 15(a)), for the three basic measurements along line A_1 :		
Translative error	is affected by angular error	and by offset
Positioning $\delta_x(x)$	pitch $\epsilon_z(x)$	B_1 to line A_2
Positioning $\delta_x(x)$	yaw $\epsilon_y(x)$	C_1 to line A_3
Positioning $\delta_x(x)$	both $\epsilon_z(x), \epsilon_y(x)$	both B_2 and C_2 to line A_4
Lateral $\delta_z(x)$	roll $\epsilon_x(x)$	C_1 to line A_4
Lateral $\delta_y(x)$	roll $\epsilon_x(x)$	C_1 to line A_3

During motion Z (Fig. 15(b)). With respect to the saddle as the carrier of the Z motion, the tool can only be offset in Y (not in X). It can also be offset in Z^* for various spindle extensions. For the three basic measurements along line B_1 and the shortest spindle extension (z_{min}^*):

Translative error	is affected by angular error	and by offset
Positioning $\delta_z(z)$	pitch $\epsilon_x(z)$	B_1 to line C_2
Lateral $\delta_x(z)$	roll $\epsilon_z(z)$	B_1 to line C_2

For longest spindle extension (z_{max}^*):

Lateral $\delta_x(z)$	yaw $\epsilon_y(z)$	B_2 to line C_1
Lateral $\delta_y(z)$	pitch $\epsilon_x(z)$	B_2 to line C_1

During motion Y (Fig. 15(c)). With respect to the carrier of motion Y, i.e., the headstock, the tool can only be offset in Z^* (extension of spindle). Therefore, with respect to the three basic measurements along line B_1 :

Translative error	is affected by angular error	and by offset
Positioning $\delta_y(y)$	yaw $\epsilon_x(y)$	C_1 to line B_2
Lateral $\delta_x(y)$	roll $\epsilon_y(y)$	B_1 to line C_2

errors in the primary way, see Fig. 15. In this figure thick arrows indicate directions of errors.

In a way similar to the above table, extreme offsets and primary effects of angular motions may be determined for other structural configurations of machine tools based on the rules given in the Appendix.

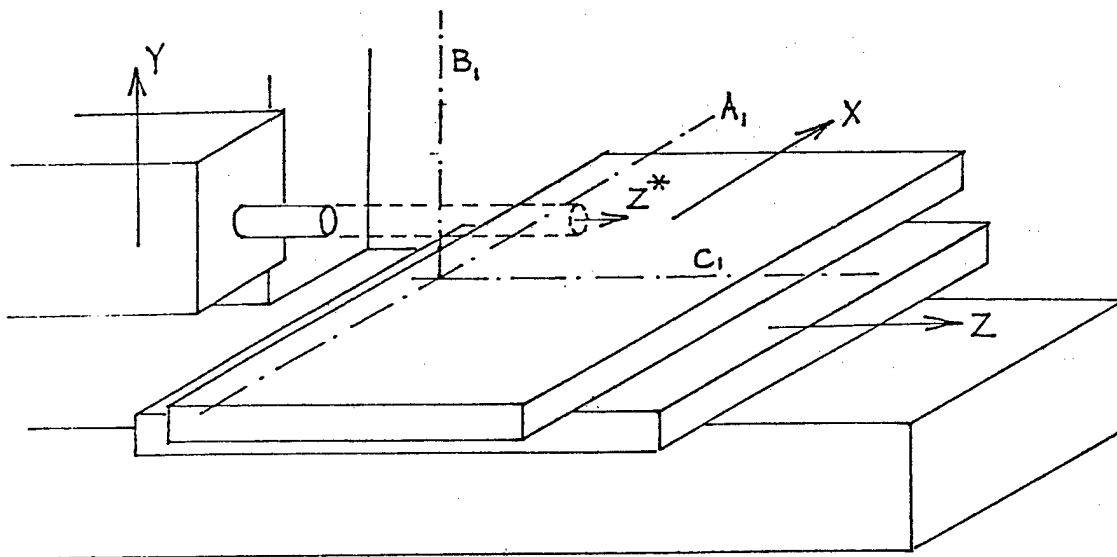


FIG. 13. Horizontal boring mill: axis and motion nomenclature.

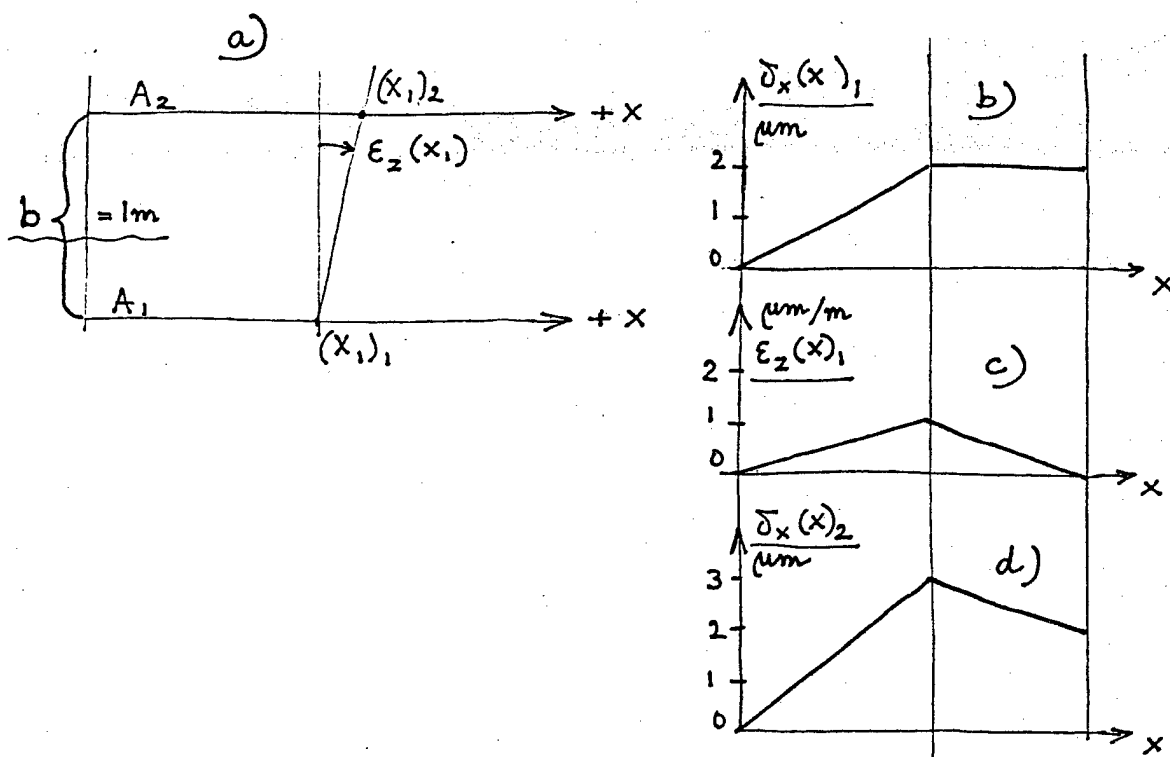


FIG. 14. Simulation of primary effects on translative motion.

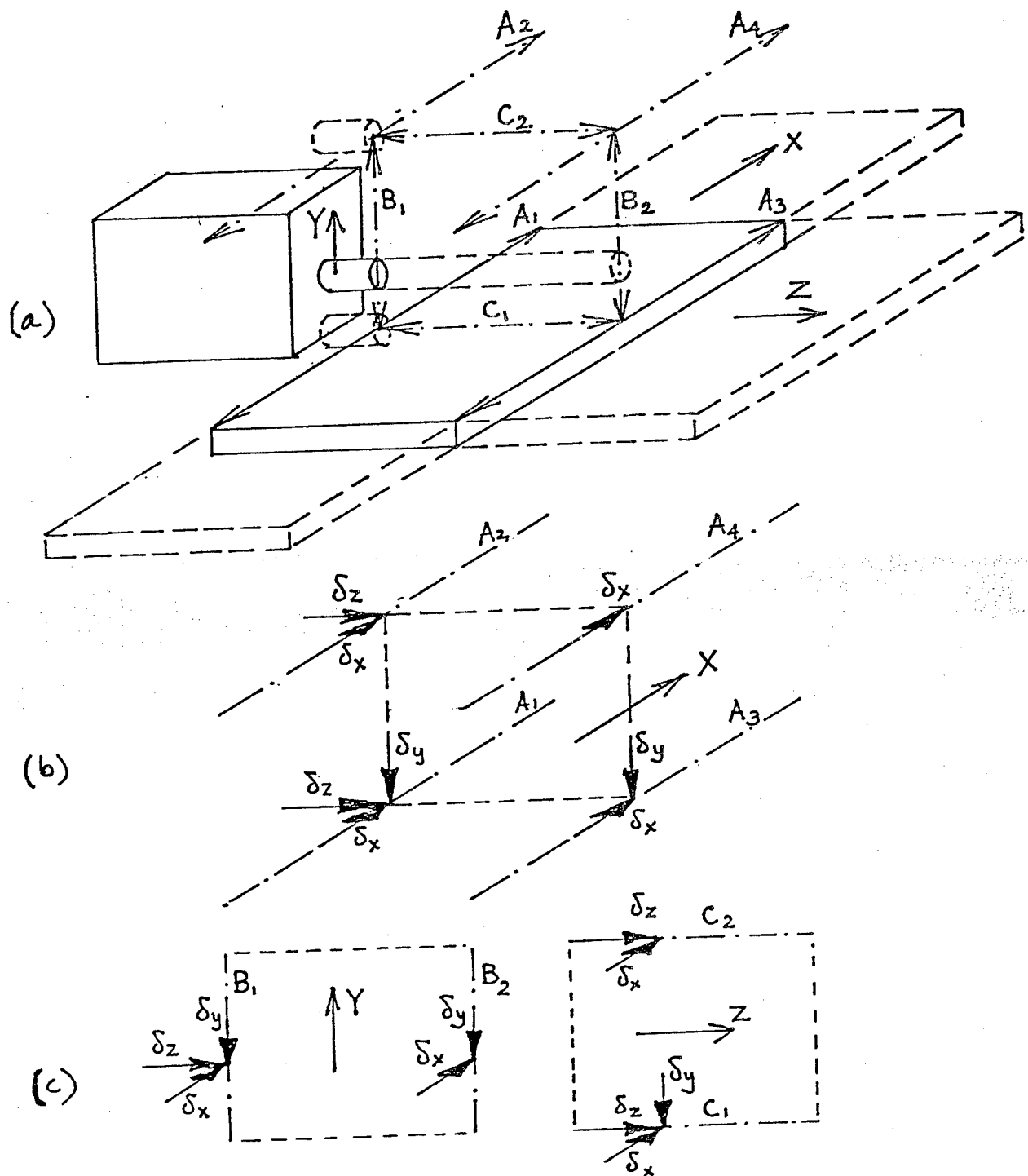


FIG. 15. Diagram of horizontal boring mill translation and offsets.

4.2 SECONDARY EFFECTS

Secondary effects are exactly specified in the Appendix and they are also specified here for the case of the horizontal boring machine. The example of the horizontal boring machine will be illustrated here. The six lateral measurements along the three basic lines A_1, B_1, C_1 are affected in a way which will be explained for the case of $\delta_z(y)$ first measured along line B_1 , Fig. 16. If this measurement was repeated along any other line $B_2, B_3 \dots B_5 \dots$ in the XY plane of the Master Part the result would be affected by the roll $\epsilon_x(x)$. The effect would apply to the directional component e_d of the error.

If the roll error were like that in Fig. 17 and line B_1 was located at x_1 the extremes of the effect would occur at locations x_2 and x_3 and they would be expressed by the differences in the error e_d , which are noted as β .

$$\beta_{1,2} = \epsilon_x(x_1) - \epsilon_x(x_2) \quad (8)$$

and

$$\beta_{1,3} = \epsilon_x(x_1) - \epsilon_x(x_3). \quad (9)$$

Consequently the $\delta_z(y)$ error would move as shown in Fig. 18. In this graph $\delta_z(y)$ is represented, for the sake of clarity, by only a line instead of an error field. The rotation of this line by $\beta_{1,2}$ and $\beta_{1,3}$ establishes itself an error field. Through the effect of the roll $\epsilon_x(x)$ the directional component e_d of the error varies between e_{d2} and e_{d3} as extremes.

It is obvious that if the extreme errors $\delta_z(y)_2$ and $\delta_z(y)_3$ satisfy a tolerance, it will be satisfied in any other position x , too.

It is obvious that from the "builder's point of view" an error of squareness of the Y motion (Sec. 3.3) to the Z motion will vary with the position X of the table and it will pass through extremes at positions x_2 and x_3 .

In the above example the lateral error $\delta_z(y)$ was affected by the roll $\epsilon_x(x)$ and it had extremes at the extremes of the roll. It could be shown that the same lateral error is also affected by the pitch $\epsilon_x(z)$ and it reaches its extremes at the extremes of the pitch.

In a similar way the effects shown in Table 2 occur (see Fig. 19a).

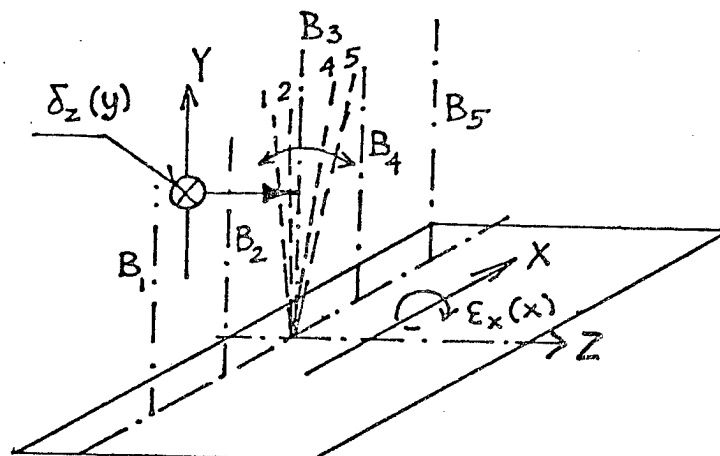


FIG. 16. Diagram of secondary effects.

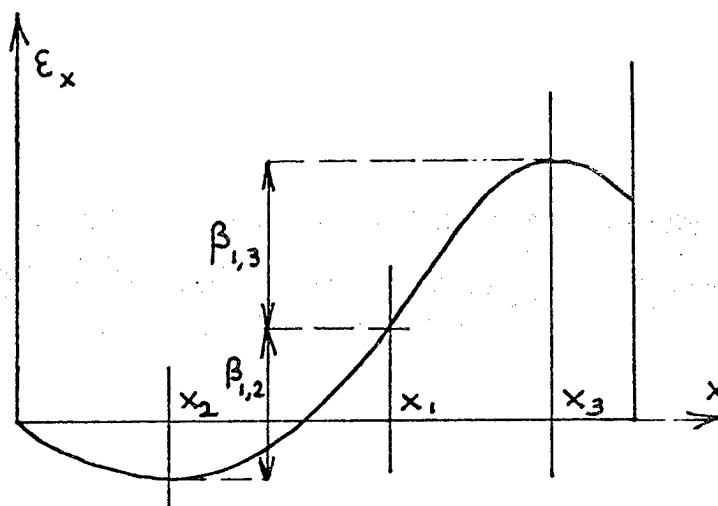


FIG. 17. Simulated roll-error recording.

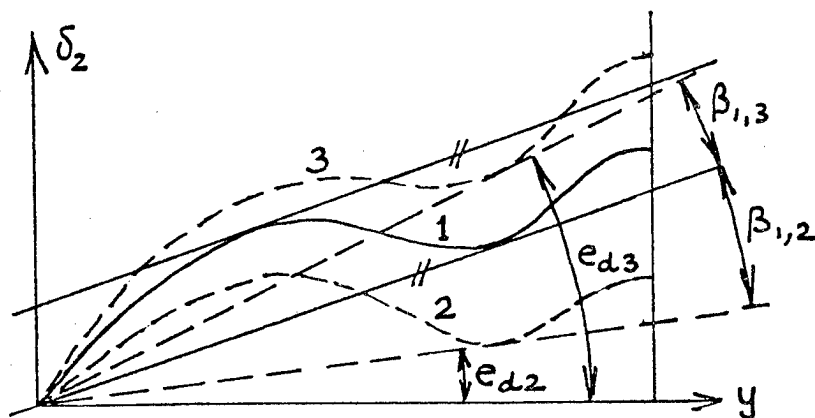


FIG. 18. The effect of roll error on $\delta_z(y)$.

TABLE 2. Secondary effects on lateral errors.

Case ^a	Lateral error	is affected by and has extremes at extremes of:
a)	$\delta_y(x)$	roll $\epsilon_z(z)$
b)	$\delta_z(x)$	yaw $\epsilon_y(z)$
c)	$\delta_x(y)$	pitch $\epsilon_z(x)$ and roll $\epsilon_z(z)$
d)	$\delta_z(y)$	roll $\epsilon_x(x)$ and pitch $\epsilon_x(z)$
e)	$\delta_x(z)$	yaw $\epsilon_y(x)$
f)	$\delta_y(z)$	roll $\epsilon_x(x)$

^aCorresponds to Fig. 19.

In cases a), b), e), and f) there are two extremes of the effect which correspond to Fig. 19. In cases c) and d), one and the same lateral error is affected by two different angular errors, each of which will have two extremes. In the worst instances (take case c) the x and z coordinates may combine so that the two maxima or the two minima of ϵ_z will coincide. Such two cases will represent the extremes of the secondary effects on $\delta_x(y)$.

4.3 SIMULTANEOUS ACTION OF PRIMARY AND SECONDARY EFFECTS

(a) It is rather easy to combine primary and secondary effects if the offset involved in the primary effect has a different direction than the coordinate motion with which the angular error causing the secondary effect is associated.

Such is, e.g., the case of $\delta_z(x)$ (see Fig. 20).

Primarily, it is affected by $\epsilon_x(x)$ and offset b in direction Y. Secondly, it is affected by $\epsilon_y(z)$ associated with motion Z.

It is possible to arrange for the extremes of the two effects independently and combine them. A corresponding evaluation will be done so that: first the extremes $\epsilon_y(z_2)$ and $\epsilon_y(z_3)$ will be determined at coordinates z_2, z_3 and the differences

$$\beta_{1,2} = \epsilon_y(z_1) - \epsilon_y(z_2)$$

$$\beta_{1,3} = \epsilon_y(z_1) - \epsilon_y(z_3)$$

(10)

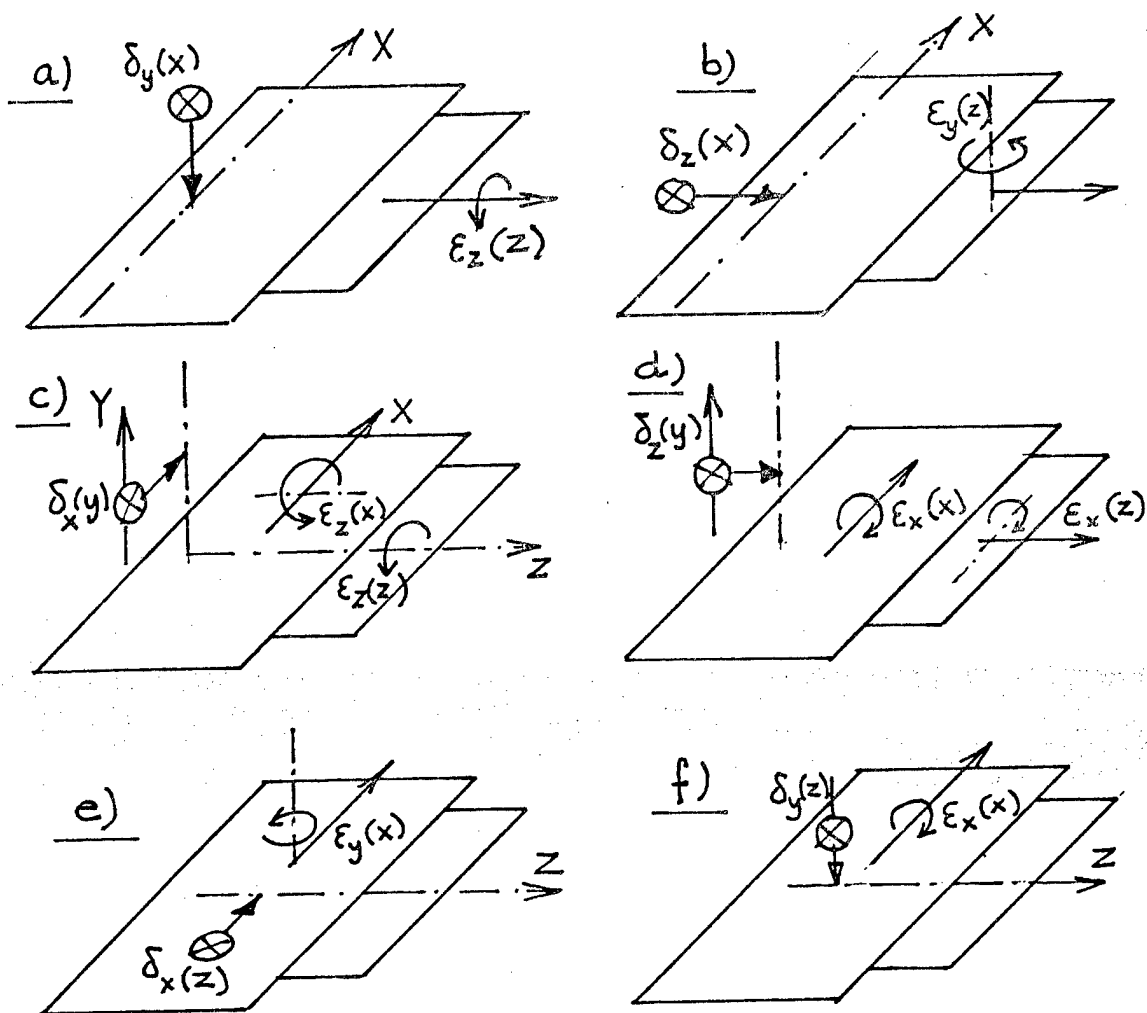


FIG. 19. Rotational and lateral motion nomenclature (secondary effects).

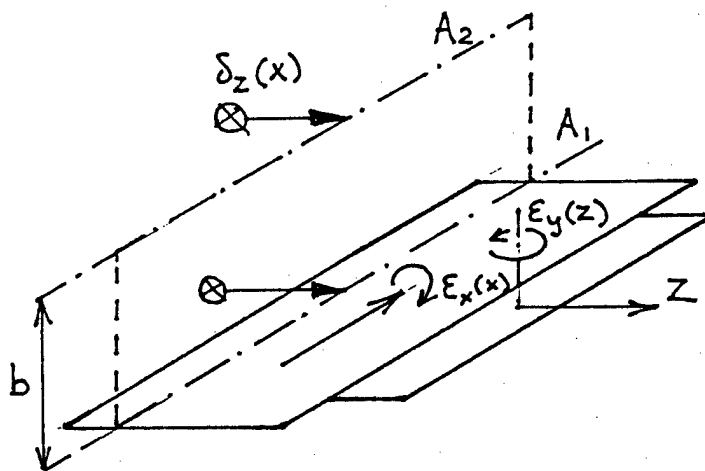


FIG. 20. Simultaneous action of primary and secondary effects.

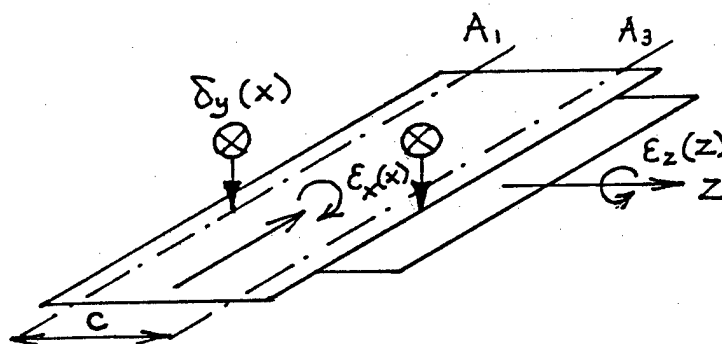


FIG. 21. Simultaneous action of primary and secondary effects (difficult case).

established with respect to the coordinate z_1 of both lines A_1 and A_2 . The measurements of $\delta_z(x)$ along these lines will be corrected by $\beta_{1,2}$ and $\beta_{1,3}$ in the way indicated in Fig. 18 and both will have to satisfy the tolerance.

(b) A difficult case is such where the directions of the offset in the primary effect and the coordinate motion causing the secondary effect coincide. Such is the case of $\delta_y(x)$, see Fig. 21. In general the extremes on lines A_1 and A_3 of the primary effect of $\epsilon_x(x)$ arise at different values z_1 and z_3 than are the values z_2 and z_4 at which extremes of the secondary effect of $\epsilon_z(z)$ occur. Consequently the extremes of the combined primary and secondary effects may occur at any other values of z than z_1, z_2, z_3, z_4 .

It is then not possible to use the approach of tolerances at extremes to guarantee satisfaction everywhere else.

Fortunately, these incompatible causes are rare as is explained in the Appendix and for the horizontal boring machine it is only the case of $\delta_y(x)$. In these cases the formally correct approach would be to express the error $\delta_y(x)$ as a function of both x and z coordinates:

$$\delta_y(x, z) = \delta_y(x, z_1) + \epsilon_x(x)(z - z_1) + [\epsilon_z(z) - \epsilon_z(z_1)]x \quad (11)$$

and to evaluate it as a number of $\delta_y(x)$ functions for a number of z values in small increments of z and check every one of them against the tolerance template.

However, as a compromise, it is much more practical to measure first $\epsilon_x(x)$ and $\epsilon_z(z)$ and, depending on their shape, decide and select two or three values of z at which to measure $\delta_y(x)$.

4.4 MEASURING THE ANGULAR ERRORS; FLEXIBLE TABLE

Assuming a rigid table the three angular errors are measured as shown in Fig. 22. The pitch and yaw are shown to be measured using the laser interferometer and the roll is measured using two electronic levels, one on the table and one on the spindle in a differential connection.

Measurements during motions Y and Z are arranged in a similar way except for the measurement of the roll $\epsilon_y(y)$, for which electronic levels cannot be

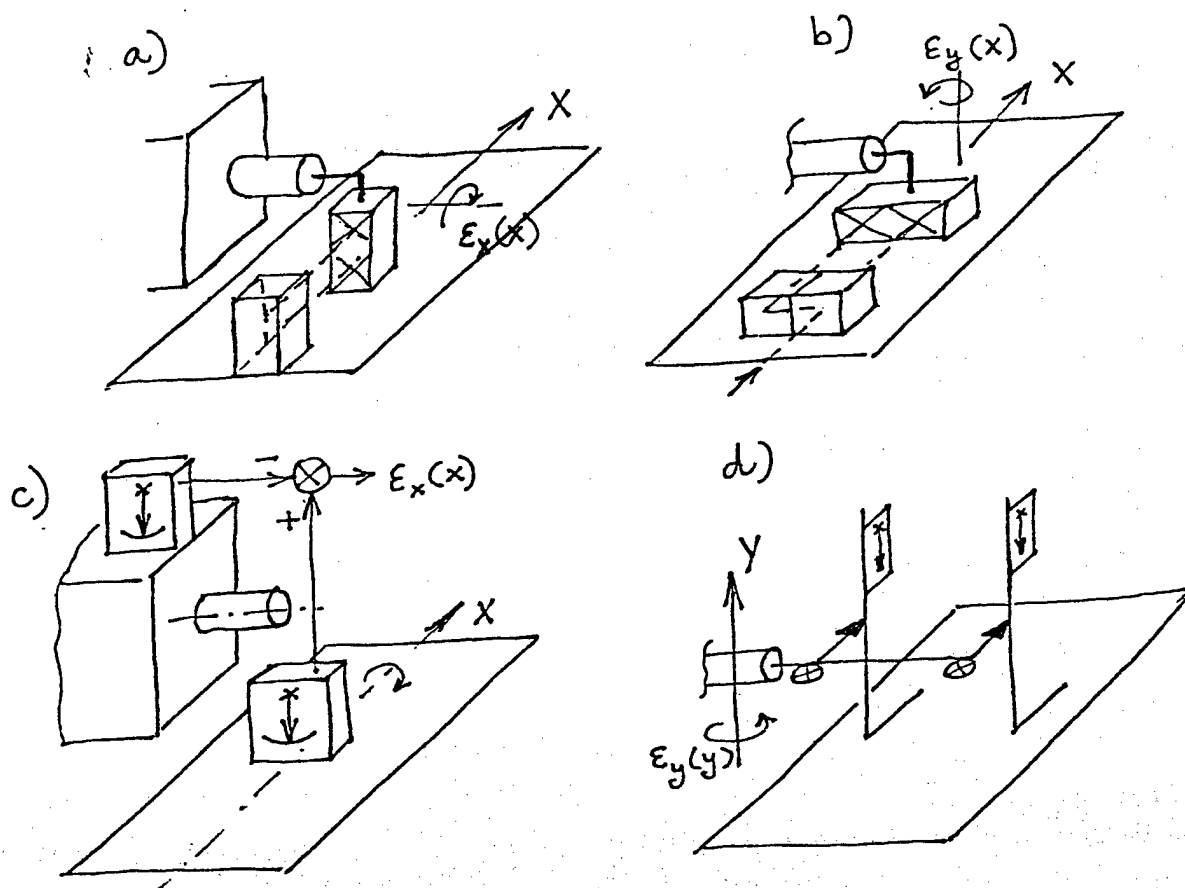


FIG. 22. Measurement of angular errors (flexible table).

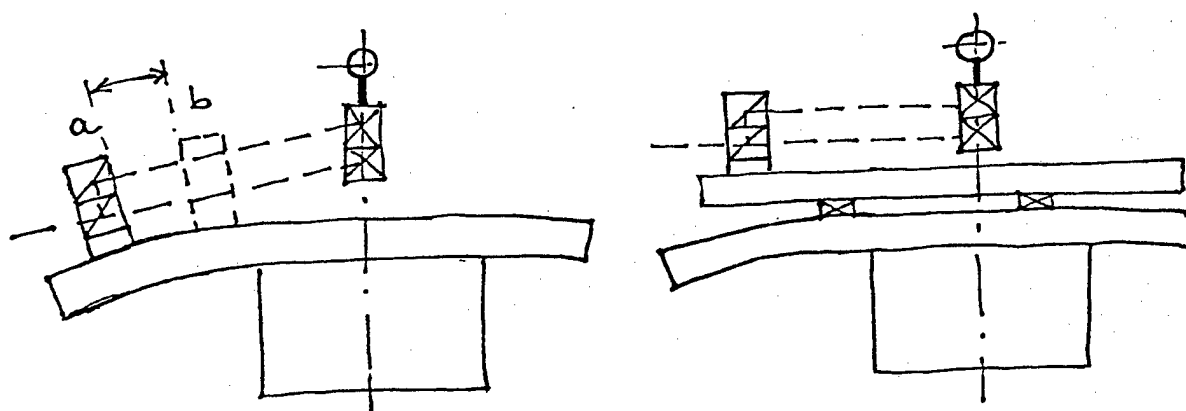


FIG. 23. Correction for table flexibility during angular error measurement.

used because it is a rotation in a horizontal plane (not out-of-the-horizontal plane as for $\epsilon_x(x)$ and $\epsilon_z(z)$). However, similarly as for the roll $\epsilon_x(x)$ the gravitational field may be used as reference and a measurement as indicated in Fig. 22(d) arranged. Straightedges C_1 and C_2 offset in Z can be set vertically using a frame level to establish their parallelity and measurements $\delta_x(y)$ carried out on them. The roll $\epsilon_y(y)$ can be derived from the difference of the two.

Flexible tables or carriers cannot be used as references for angular error measurements. As illustrated in Fig. 23 the interferometer located in position a on a sagged part of a table will measure the pitch $\epsilon_z(x)$ quite differently than the interferometer in position b. It is necessary to locate the interferometer on a beam supported on specified reference points of the table so as not to be affected by the deformation of the table.

The angular errors arise from similar effects as the errors of straightness of motion and they sometimes show a similar dead zone as was illustrated in Fig. 7. Therefore, they should be measured according to Rule 1⁺ which is identical with Rule 1, Sec. 3.5 except that the words "straightness measurement" are replaced by "angular error measurement."

4.5 TOLERANCING AND EVALUATING IN A 3-D SPACE

For an exact correlation of the accuracy of the workpiece with the motions of the machine tool it would be necessary to first introduce two coordinate systems, one for the workpiece and one for the machine tool and write the equations relating the two together. However, except for such cases where the coordinate z of the workpiece depends both on the motion of the saddle and of the spindle extension, the relationships are straightforward. We will therefore equate the corresponding coordinates. On the workpiece we have axes X , Y , and Z . The latter corresponds, on a horizontal boring machine, Fig. 1, to the saddle motion which, according to the existing standard should be denoted W while Z is used for spindle extension. We are denoting the saddle motion Z , and the spindle extension which has no other meaning than a shift of the origin of Z , will, if at all necessary, be denoted Z^* .

Workpiece as a Uniform Assemblage of Points

The approach discussed here has been developed in Ref. 33 and it is briefly recapitulated in the following.

The general representation of a workpiece is as one assemblage of points in the three-dimensional working space of the machine, see Fig. 24. We will be concerned with the distance between any two points 1 and 2 in this space. In an absolute way, the actual position of a point is determined by the deviations $\delta_x, \delta_y, \delta_z$ from its ideal position. Let us determine the distance of the two points by means of its three coordinate components as

$$(x_2 - x_1), (y_2 - y_1), (z_2 - z_1) \quad (12)$$

and the component errors of the distance as

$$(\delta_{x2} - \delta_{x1}), (\delta_{y2} - \delta_{y1}), (\delta_{z2} - \delta_{z1}). \quad (13)$$

The component errors (Eq. (13)) can be considered dependent each on the total distance of the two points as expressed by the components (Eq. (12)). There are good reasons for this choice of expression.

One reason is practical, considering the generation of errors. Moving from point 1 to point 2 on a machine tool consists of sections 1 - 1' in the direction X, 1' - 1" in the direction Z and 1" - 2 in the direction Y. Considering one of the component errors, e.g., δ_x this is generated in parts as: the positioning error $\delta_x(x)$ over the distance $(x_2 - x_1)$, the lateral error $\delta_x(z)$ along the distance $(z_2 - z_1)$ and the lateral error $\delta_x(y)$ along the distance $(y_2 - y_1)$. In an analogous way the errors δ_y and δ_z are generated in all three component motions.

The other reasons are related to various practical aspects of tolerancing workpieces (see Fig. 25) where dimensions A and B of flat surfaces are considered. It may be expected that the tolerance B will be greater than A because the surface length of B is longer than A. In this instance surface length is height.

The relationship between position error and distance as a generalization of the relation indicated in Eq. (3) is expressed as:

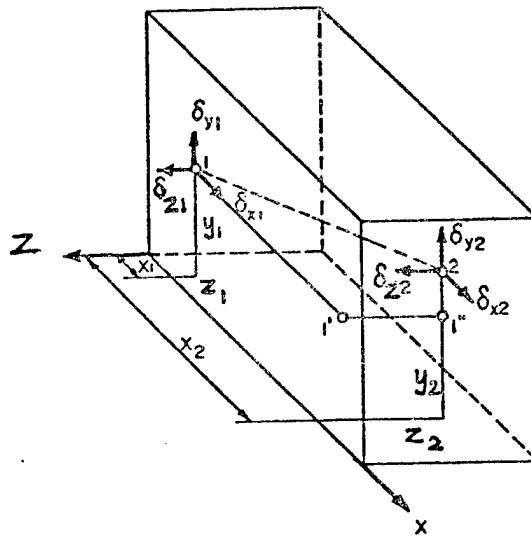


FIG. 24. Representation of 3-D working space.

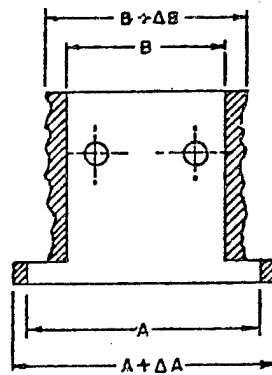


FIG. 25. Tolerance depends on workpiece shape.

$$|\delta_{m1} - \delta_{m2}| \leq A_{mx} + K_{mx}|x_2 - x_1| + A_{my} + K_{my}|y_2 - y_1| + A_{mz} + K_{mz}|z_2 - z_1| \quad (14)$$

where $m = x, y, z$.

The interpretation is as follows: The relative component errors of any two points are permitted to be equal to a constant value plus a sum of values proportional to the coordinate component distances of the two points. The constants A_{mn} and K_{mn} may be chosen with respect to the significance of the individual directions.

The error consists of three parts which have the same form as Eq. (3):

$$|\delta_m^{(n)}|_{1,2} = A_{mn} + K_{mn}|n_2 - n_1| \quad (15)$$

where $m = x, y, z$ and $n = x, y, z$; for $m = n$ it is the positioning error and for $m \neq n$ it is the lateral error.

From this concept of the workpiece results the following rule for tolerancing and evaluating of machine tool motions errors:

Rule 4. Any translative error measured along any line of motion in the working space has to satisfy a corresponding tolerance rule of Eq. (15). The constants A and K of the tolerance template may be chosen different for different directions of motions and for different parts of the working space.

The latter part of the rule may be interpreted so that tolerances may be tighter in a preferred part of the working space such as its central part or its frontal part.

It is obvious that in this approach the angular errors (pitch, roll, yaw) are not evaluated on their own. Their effects are included in the evaluations according to Rule 4. Accordingly, this rule may then be refined as follows:

Rule 5. Evaluation according to Rule 4 applies to measurements in which the individual translative errors are extremely affected by angular motions as illustrated in Table 1 and Table 2 and as specified in the Appendix.

4.6 THE TOTAL OF NECESSARY MEASUREMENTS

There are two ways to arrange the tests:

1. The first one follows directly from Rule 5 and in it all the individual translative measurements to be evaluated are carried out.

Taking the example of the horizontal boring machine, the total of such measurements is shown in Fig. 15, where the thick arrows on the individual lines indicate the directions of errors to be measured along these lines. This constitutes 18 individual measurements. That is obviously a large number of measurements and, actually, additionally 6 angular error measurements (per Table 2, Sec. 4.2) are necessary and evaluations for secondary effects as described in Sec. 4.3 have to be carried out. Some of the translative measurements are difficult and they necessitate fixing the interferometer or straightedge high above the table.

2. Therefore, another more practical approach may be chosen. It consists of basically measuring the three translative errors each along one line only and the three angular errors for every coordinate motion and deriving from them all the translative errors to be evaluated. Thus, instead of the $18 + 6 = 24$ measurements of the preceding method, only $3 \times 6 = 18$ measurements are sufficient and the rest is replaced by a rather modest computing effort. Actually, only 17 measurements are needed because $\epsilon_z(y)$ does not affect any translative errors. In deriving the 18 errors to be evaluated from the 17 measurements, formulas (A-1) and (A-3) of the Appendix are used. The main advantage of this method consists in the elimination of the rather tedious arrangements for locating the laser interferometer and the straightedges at the various positions needed in the first approach.

In practice any combination of these two methods is possible.

Another, still simpler but less rigorous, approach consists in not computing translative errors over the whole working space, but measuring them along single lines passing through the center of the working zone as in Ref. 31; measuring also the angular errors and imposing on them tolerances separate from the tolerance for the translative errors. An example for determining the size of such a tolerance is given in Fig. 26. If, e.g., the tolerance template for the $\delta_z(x)$ measurement along the center line A_1 , Fig. 26(a), is given as shown in Fig. 26(b) by T_1 , we may decide to admit that the errors along line A_2 at the top of the work space are to be double that along A_1 , and create a template T_2 . This would lead to a tolerance

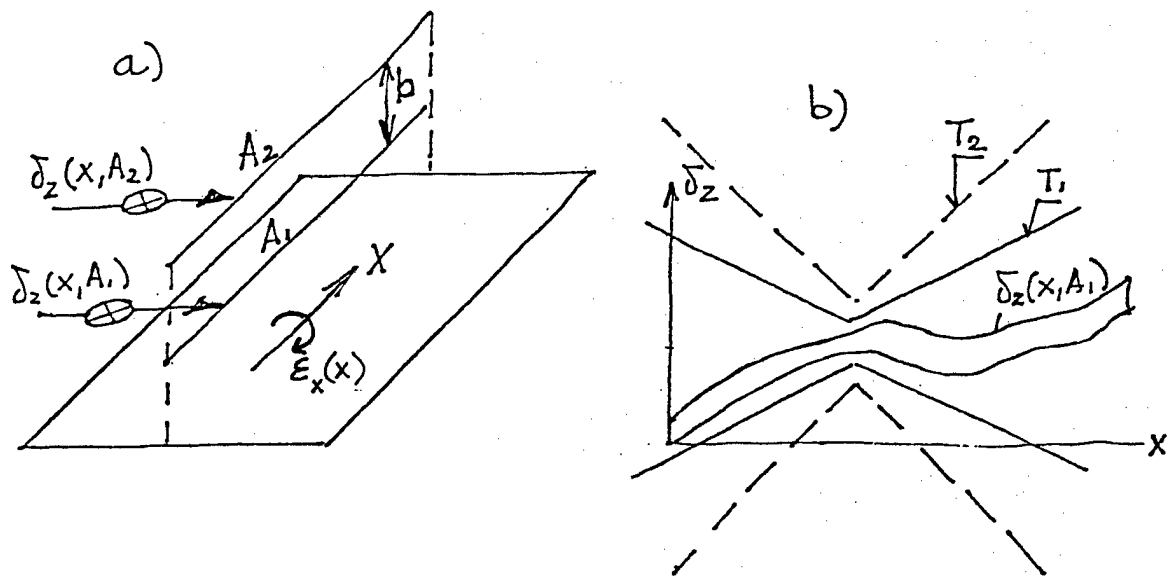


FIG. 26. Application of tolerance template to work zone.

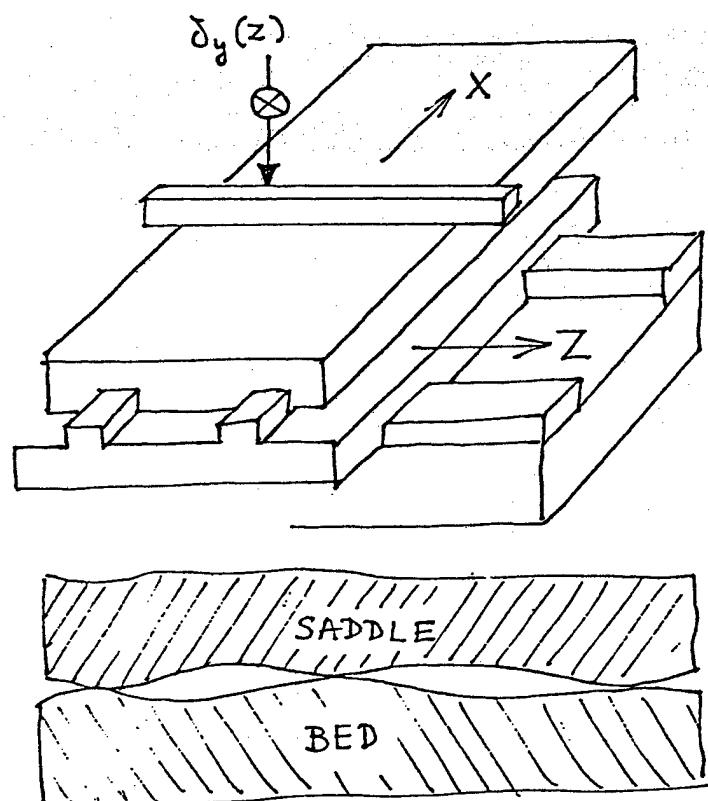


FIG. 27. Slide deformations that affect slide motion $\delta_y(z)$.

template T_r (not shown) for the roll $\epsilon_x(x)$ $T_r = T_1/b$, where b is the offset in Y between A_1 and A_2 . Expressed by formulas it is:

$$\begin{aligned} T_1: \quad |\delta_{z1} - \delta_{z2}| &< A_{zx} + K_{zx}|x_1 - x_2| \\ T_r: \quad |\epsilon_{x1} - \epsilon_{x2}| &< \frac{A_{zx}}{b} + \frac{K_{zx}}{b}|x_1 - x_2|. \end{aligned} \quad (16)$$

5.0 EFFECTS OF DEFORMATIONS OF THE STRUCTURE ON SUPERPOSITION OF MOTIONS

So far, it has been assumed that the individual six errors associated with each individual coordinate motion depend on the motion only during which they were measured and that the effects of the individual motions may be superimposed.

In reality this is only so to the extent that there is a certain amount of mutual cross-influence between errors associated with different motions. Let us, for example, consider the case of the motion Z of the saddle carrying a table executing the motion X and let us concentrate on the out-of-line error $\delta_y(z)$. The detail at the bottom of Fig. 27 represents a possible form of contact between the saddle and the guideway on the bed. The form of this contact influences the error $\delta_y(z)$. If the saddle is not rigid enough it can deform under the weight of the table and the bed can deform as well as changing the form of the contact and affecting $\delta_y(z)$. The deformation will be different depending on the position of the table along the X axis. In this way $\delta_y(z)$ will depend on X .

We are assuming that effects of this type of deformation are small and they are limited to compound slides of the type just discussed. There is practically no evidence available about the significance of these effects. Here we neglect them. In cases where they could not be neglected the measurements like $\delta_y(z)$ above would have to be repeated for various locations of the table and some kind of statistical approach would have to be chosen.

6.0 WEIGHT EFFECTS

Deformations caused by the weight of moving bodies affect the accuracy of the machine. These moving bodies are parts of the structure (tables, saddles, headstocks) and the workpiece. These effects and ways how to decrease, compensate for or eliminate them have been described in detail in Ref. 24, Sec. A5. Those caused by the moving parts of the structure are included in all the measurements of translative and angular errors. The effect of the weight of the workpiece may, for instance, be such as shown in Fig. 28. In the extremes of the X travel the bed of the machine may be twisted and this will affect some of the measurements and especially $\delta_y(x)$ and $\delta_x(y)$. The question is how to assess the effects of all the possible workpiece weights.

The answer is analogous to the case of effects of offsets and angular errors on translative errors. The effect of any workpiece weight may be interpolated between that of zero weight and that of maximum specified weight. If evaluation of accuracy is based on checking against a tolerance then the rule may be stated:

Rule 6. For workpiece weight effects a check must be made both without a workpiece (or with a very light one) and with a workpiece of maximum specified weight. If the tolerance is satisfied for both these extremes it will be satisfied for any intermediate workpiece weight.

Not all errors of motion are affected by workpiece weight. For a given machine tool structure it can be rather easily estimated which of the translative and angular errors may be affected and those then have to be carried out two times.

7.0 THERMAL EFFECTS

The effect of thermal deformations of the structure on the accuracy of the workpiece may be very significant. Such effects may be divided into those caused by sources internal to the machine, by the environment and by the cutting process. We will neglect those of the cutting process by assuming that we are concerned with the accuracy of the finishing operation in which the power of the cutting process is small and that care is taken in insulating

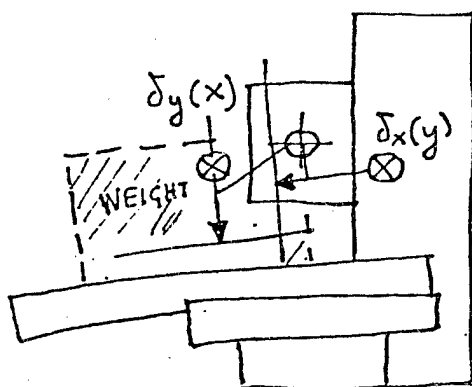


FIG. 28. Weight deformation and its effects on $\delta_y(z)$ and $\delta_x(y)$.

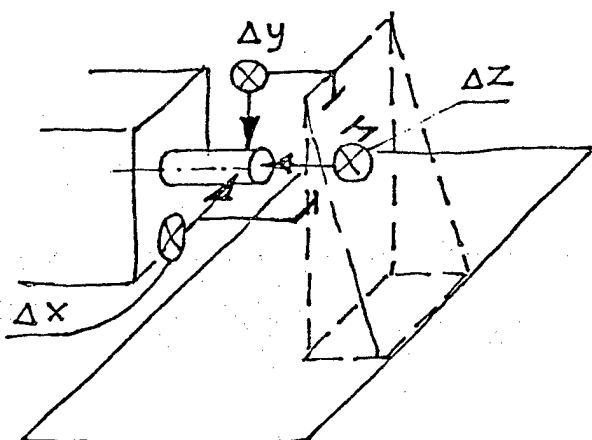


FIG. 29. Environmental effects Δx , Δy , and Δz .

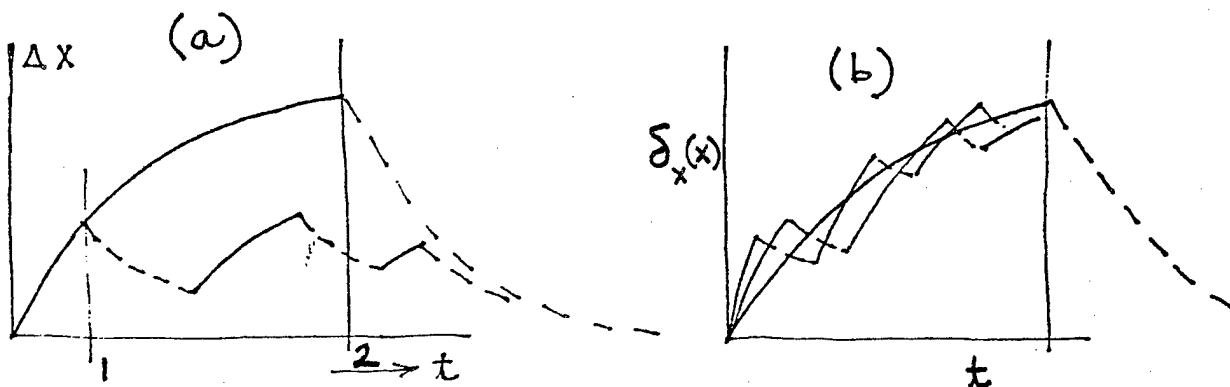


FIG. 30. Thermal motion vs operating cycle.

the contact of the fluid and of the chips with the structure. The effects of the environment and those internal to the machine can be measured separately and then superimposed. However, it is more usual to impose separate tolerances on them.

7.1 ENVIRONMENTAL EFFECTS

The effect of the varying temperature of the surrounding air is often not negligible and the usual shop air-conditioning system with high local air flow does not often help much. A rather simple but very good method of testing for it was suggested by Bryan. It consists (see Fig. 29) in letting the machine rest for 24 hours in a selected "typical with an inclination to extreme" setting of the positions in the individual coordinates and reading (or recording) the maximum drifts Δx , Δy , Δz between the spindle and the workpiece. In the depicted example, coordinate values were chosen so as to have a rather short spindle extension corresponding to the most often used one and with spindle at half Y travel (slightly higher than perhaps the most frequent Y used). The angle plate is considered a part of the machine.

Tolerances have to be specified for Δx , Δy , Δz . Usually it is required that all of them be well below the errors permitted which are induced internally. It is suggested that

$$\Delta x = A_{xx}/2, \Delta y = A_{yy}/2, \Delta z = A_{zz}/2$$

where A_{xx} , A_{yy} , and A_{zz} are the basic, no-motion parts of the tolerance templates for positioning.

7.2 EFFECTS OF INTERNAL SOURCES

These effects and their measurements have been discussed in detail in Ref. 27, of which this is a condensed presentation. There are various sources of heat in the machine tool. A rather detailed discussion of them was presented in Ref. 24, Sec. A7. Some of them are constant, like the heat generated in a hydraulic power source with constant delivery. Others are variable, like the heat generated in the spindle bearings, the heat in the

gearbox of the headstock, the heat ventilated out of the electric motor driving the spindle, and the heat generated by friction between the nut and the leadscrew.

Each of these sources will affect some of the measurements of translative and angular errors. It is usually rather easy to estimate which of the effects will apply.

Taking our standard example of the horizontal boring machine we may have the following situation:

(a) A hydraulic power pack and reservoir might be attached to the column affecting practically all the measurements associated with the Y motion.

(b) The spindle bearings, gearbox, and electric motor affect the headstock and the column and, consequently, they affect the measurements associated with the Y motion.

These sources are variable and depend on spindle speed. The higher the speed the stronger the effect. A continuous run with a certain speed maintained until the drifts balance (until the drift rate decreases below a specified level) followed by a complete stop for the same time as the run to balance produces a range of effects which is the extreme of ranges of effects of any intermittent run at that speed. This is illustrated in Fig. 30(a), where full lines represent running, dotted lines represent standstill, and all the curves represent a thermal drift of a certain point on the structure. A certain speed may be chosen at which a continuous run will represent the extreme condition and it will be considered equivalent to an intermittent run at any higher speed (see Fig. 30(b)); the higher the speed the shorter the running intervals. It can be so that the thermal effect reverses itself. As shown in Fig. 31 the heat of the spindle may first affect the headstock and cause a drift left with a certain time constant. It will then affect the column, producing a stronger drift right with a longer time constant. It is seen that the maximum of the effect is reached some time after stopping the spindle.

(c) The leadscrew may be a part of the feedback in such a way that the synchro or digitizer is driven from the leadscrew. It may not be so and a linear Inductosyn may be attached between the saddle and table or a measuring rack and pinion. In the former case the heat generated by friction between nut and leadscrew will cause an elongation of the leadscrew. For the recirculating ball leadscrews this heat is rather small and it does not really affect the surrounding structures. Therefore, it will not affect any other measurement than positioning.

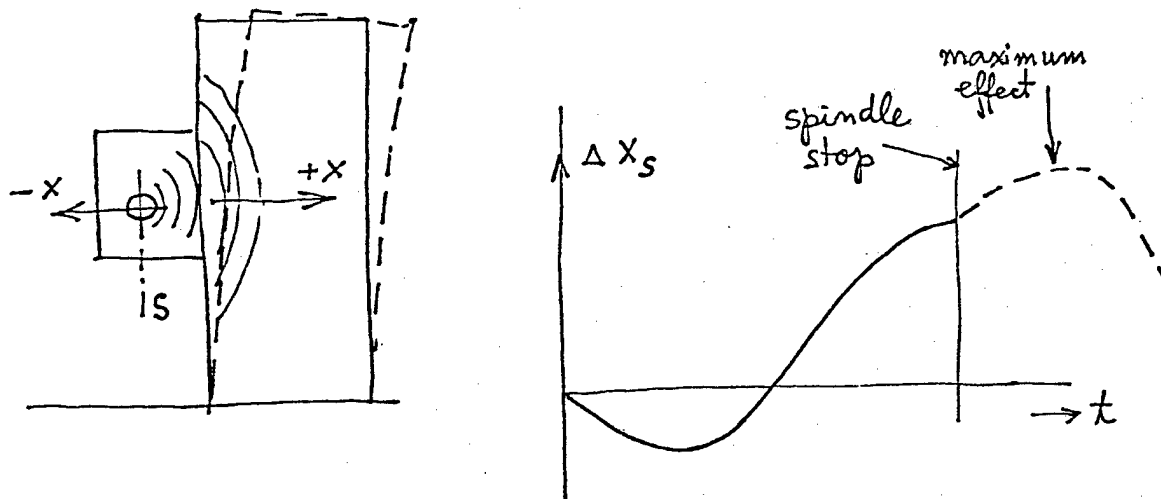


FIG. 31. Reverse thermal effect.

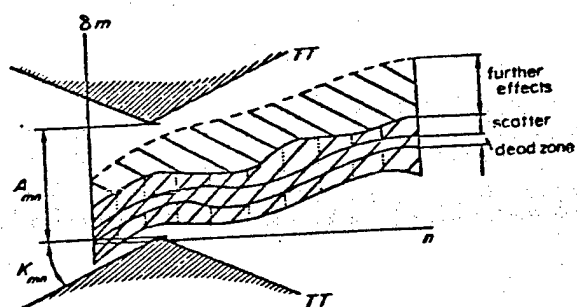


FIG. 32. Summation of all errors within tolerance template boundaries.

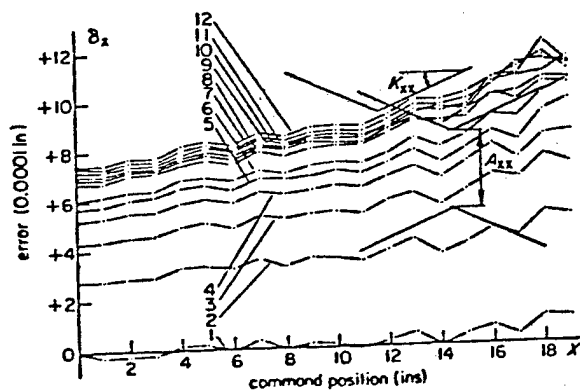


FIG. 33. Actual measurement results (summation of errors).

This effect may be tested separately from effect (b). This heat source is variable and the exercising of rapid traverse with selected intermissions will represent an extreme case.

Correspondingly, one test would be arranged with the constant source (a) switched on and the spindle source (b) continuously running at the selected speed. During this test measurements associated with the Y motion-- $\delta_y(y)$, $\delta_x(y)$, $\delta_z(y)$, $\epsilon_x(y)$, $\epsilon_y(y)$ --should be repeatedly made until the machine equilibrates and then again for a period after stopping the spindle.

Another series of tests would be arranged in an analogous way for exercising separately the X, Y, Z motions until equilibrium is reached while repeatedly measuring the positioning errors $\delta_x(x)$, $\delta_y(y)$, $\delta_z(z)$.

It should be understood that all the measurements of every translative or angular error as repeated through the whole range of the thermal state of the machine represent an error field which should, as a whole, fit into the corresponding tolerance template (see Fig. 32). An example of an actual measurement result is given in Fig. 33 as a record of the positioning error $\delta_x(x)$. The individual lines represent measurements in 15-min. intervals up to record 9, then in 30-min. intervals for a machine with a hydraulic table drive using hydraulic gages for feedback. It is seen that the change is very large between records 1 and 2 and it decreases gradually between subsequent records. It could now be agreed that having here a case of a constant heat source, a warm-up period of 30 min. is used which eliminates the changes between records 1 and 3. Furthermore, it could be agreed that once per hour the table will be driven against a reference and a zero shift used. This will mean that, e.g., the left-hand end of the record will be brought back where it was an hour ago. Consequently, the largest one-hour span (neglecting records 1 to 3) will determine the error field. This field has then to fit into the tolerance template.

The most common problem is the one of thermal effects associated with the Y headstock travel on a horizontal boring and milling machine or a machining center. The example in Fig. 34 represents the measurements which would have to be repeated through the spindle thermal cycle.

Instead of repeating these measurements throughout a thermal cycle a simplified arrangement is possible where only the end points of these measurements are repeatedly checked (see Fig. 35). The spindle runs

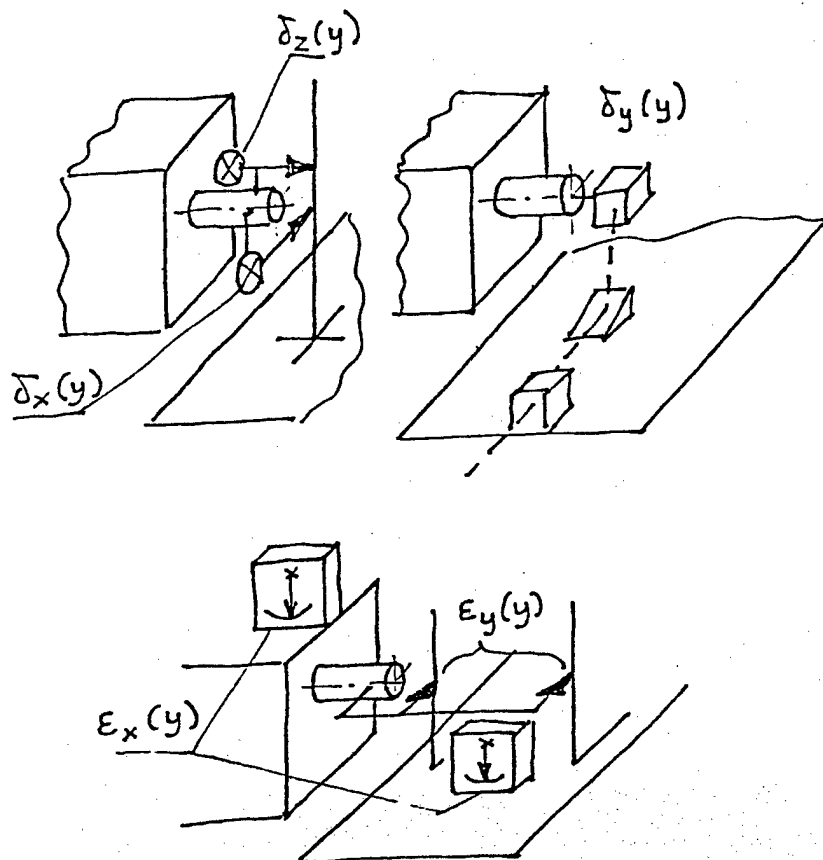


FIG. 34. Headstock measurements required to identify thermal distortions.

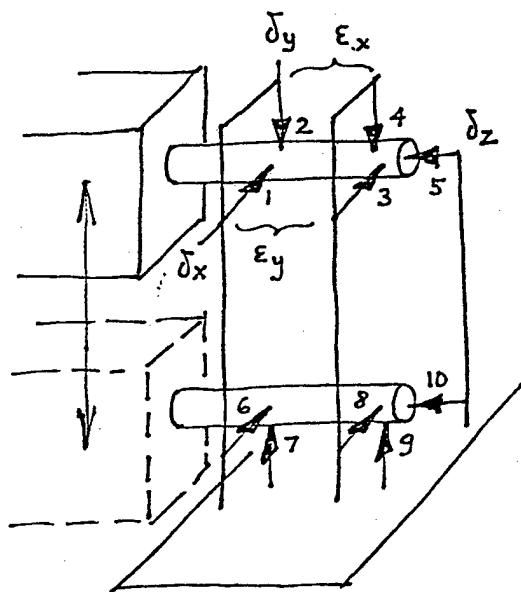


FIG. 35. Headstock measurements at the endpoints of travel.

continuously, but every, say, 15 min. its run is briefly interrupted and it is moved to both travel ends, where gages read the $\delta_x, \delta_y, \delta_z$ displacements of the mandrel at two sections differently distant from spindle end. Full records of $\delta_x(y), \delta_y(y), \delta_z(y), \epsilon_x(y), \epsilon_y(y)$ are taken either only at the beginning or at the end of the cycle and they are subsequently interpolated to the measured intermediate end positions.

The preceding illustrations should be helpful in understanding the following rule for testing for thermal effects.

Rule 7: Assess the constant and variable heat sources in the machine and estimate which of all the translative and angular errors may be affected and how they are correlated.

Determine thermal cycles for the individual variable sources, or for typical combinations of one with another, and also for constant sources.

Repeat selected measurements throughout the selected thermal cycles and establish range fields of errors during these cycles and correct them for warm-up periods and for zero-shifts if applicable. Simplified arrangements like that of Fig. 35 are acceptable.

These whole error fields have to fit into corresponding tolerance templates.

8.0 EXAMPLE OF A COMPLETE TEST SPECIFICATION

8.1 HORIZONTAL BORING AND MILLING MACHINE; HORIZONTAL SPINDLE MACHINING CENTER

In this example we will be considering a machine tool with basically the configuration as discussed before and depicted in Fig. 1. The table performs the X motion and it is mounted on a saddle performing the Z motion (reasons for not calling this axis W were explained in Sec. 4.5), the headstock moving vertically on the column--the Y motion. The motion of the spindle in and out as encountered on horizontal boring machines will be called Z*. It is not considered as a working motion for machining but as a setting motion. In the machining center it is represented by various tool lengths.

The machines where the Z motion is carried out by the column, or machines where the X motion is carried out by the column, are not discussed in detail here. The difference with these would be mainly in the effects of angular motions.

The distinction between the two types of machine tools given in the title of this section is the form of the table, which is rectangular (longer in X than in Z) for the former type (type A) and square and indexable for the latter type (type B).

The Working Space

First, it is necessary to determine the Working Space. In Fig. 36 we assume for type A the space over the table area as the Working Space. This space is mostly located in its front position and most of the machining is done on its front face using the shortest tool extension L_1 . Some machining is done with the intermediate extensions L_2 and only very deep holes would need the maximum extension L_3 . In this latter case the space will move from a rear position R over the whole travel Z_{max} . Although it is not necessary, there may be some advantage in locating lines A, B, and C for measurement of translative errors through the center of the most-used front part of the Working Space. In this case, evaluation of errors along lines other than A, B, and C may suffer from inaccuracies, caused by the measurement of angular motions (pitch, roll, yaw).

For the type B machine tool, machining is done only in the front half of the space above the table surface. The other half moves to the front after indexing by 180° . Thus, the working range of the Z coordinate extends over only half of the table width.

The type B machine may have other additional coordinate motions, like the rotation of the rotary table or its swivel around the X axis. These will not be considered here.

Reference: Motions

Let us choose not to use the table surface as reference and not to accept the table as sufficiently rigid. As indicated in Fig. 37(a) there will be a rigid plate (rigid enough to carry straightedges, laser interferometer, etc.,

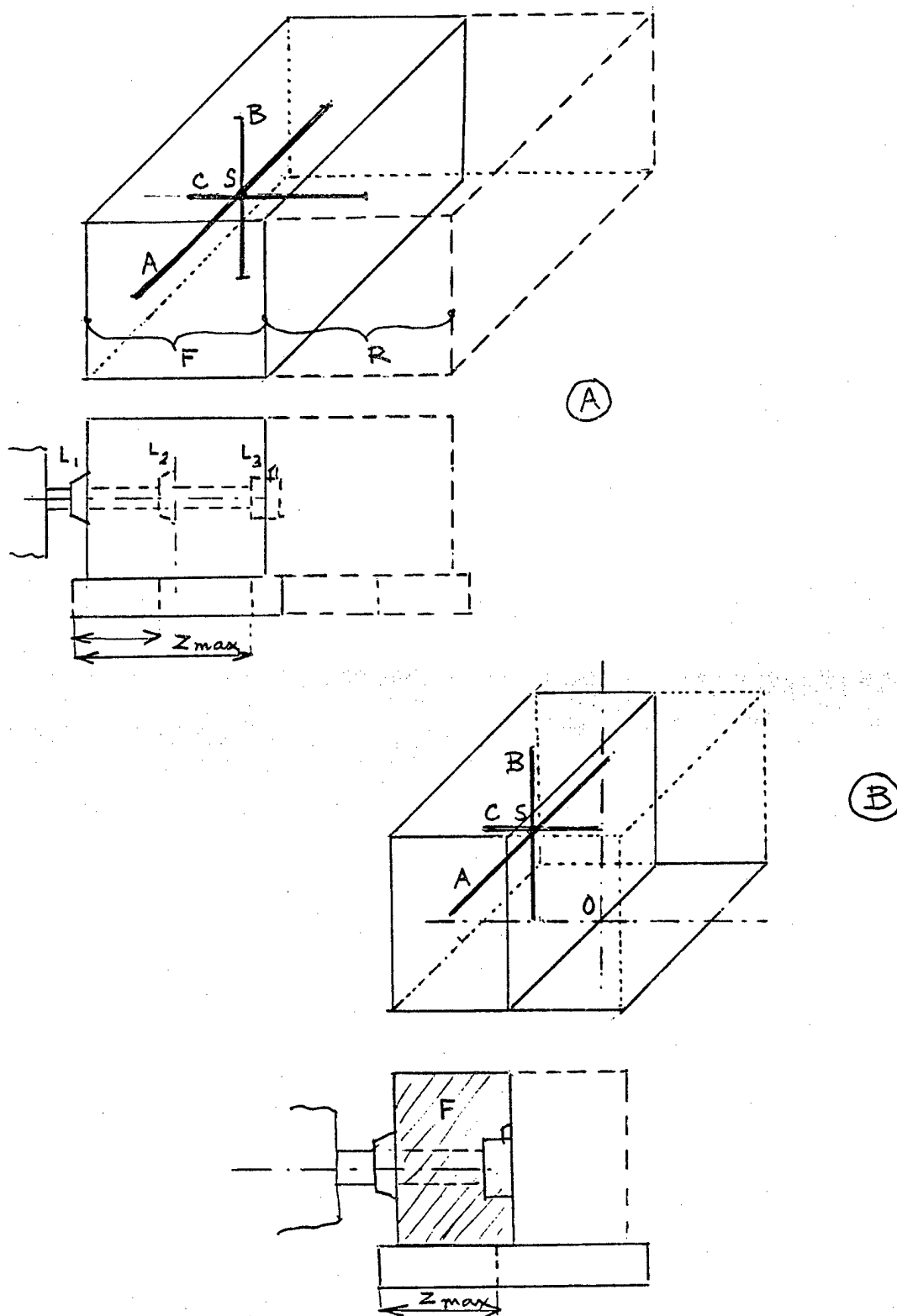


FIG. 36. Workspace description.

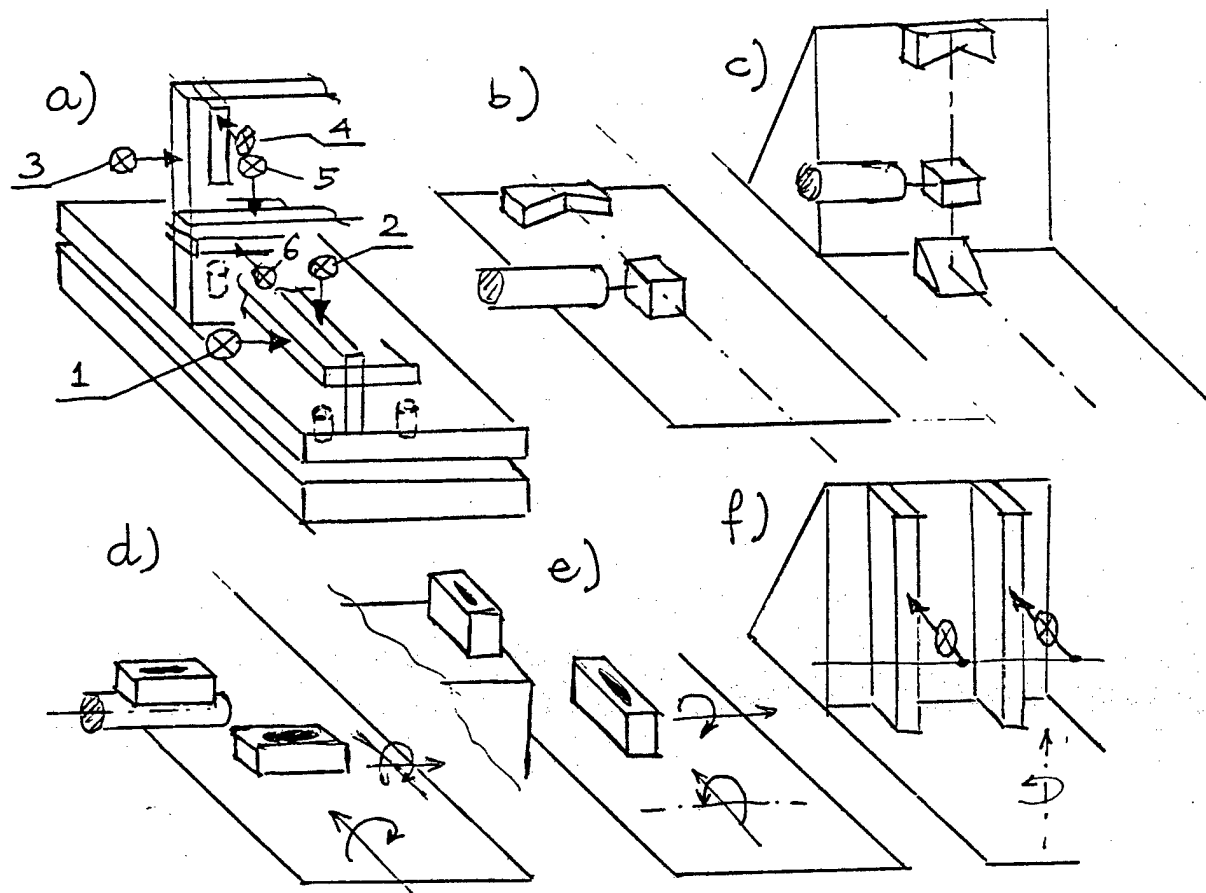


FIG. 37. Measurement arrangements for complete test for horizontal boring machines and milling machines.

without distortion) supported on three points on the table so as also not to move during accelerations of the table. The various measuring instruments--like straightedges in (a), laser interferometer straightness measuring option in (b) and (c), levels in (d) and (e), pair of straightedges for $\delta_y(y)$ measurement in (f), etc.--will be attached to this base plate and aligned using the various methods as described in Sec. 3.3 and Fig. 6.

For the reference alignment let us choose motion X as primary and motion Y as secondary. Consequently, measurements 1 and 2 in Fig. 37(a) of straightnesses $\delta_x(x)$ and $\delta_y(x)$ will be set to minimum overall readings, i.e., for zero error of direction e_d . Notice that if the lines chosen for these measurements (line A in Fig. 36) were located elsewhere in the Working Space and aligned for zero e_d in $\delta_z(x)$ and $\delta_y(x)$, such alignment would differ from that in line A and measurements $\delta_z(x)$ and $\delta_y(x)$ along A would not result in zero e_d . Thus, we are aligning only line A with motion X.

The vertical straightedge along line B will be adjusted for zero e_d in measurement 3 of $\delta_z(y)$, but the straightedge for measurement 4 of $\epsilon_x(y)$ has to be set square to straightedge 2.

The straightedges along line C have to be set so that 5 is square to 3 and 6 is square to 1.

Wherever we spoke here about adjusting or setting straightedges in various exact directions it was meant figuratively. In reality, the method as described in Sec. 3.3 using a precision square as an aid will be applied.

Selection of Measurements

Let us decide that the translative measurements will be measured along one line (in two perpendicular directions) in the Working Space each, i.e., here along the lines A, B, and C. This represents six measurements of straightness and three measurements of positioning. For the measurements along line C maximum spindle extension will be used and along lines A and B the shortest one.

Translative errors along other lines which are located at extreme offsets will be evaluated from the above-mentioned nine translative measurements and from the corresponding angular measurements which cause primary and secondary effects, as explained in Secs. 4.1 and 4.2 and defined in the Appendix.

All the measurements and combinations of effects are summarized further below under "Construction of Translative Errors." All angular error measurements will be needed except $\epsilon_z(y)$, in total eight angular error measurements.

Evaluation of tolerance will be carried out for a total of 18 "constructed" translative errors.

Methods of Measurement

The translative errors will be measured as it was explained in Secs. 3.1, 3.2, 3.5, 3.6 and stated in Rules 1 and 2.

The angular errors will be measured as explained in Sec. 4.4 and stated in Rule 1⁺.

Evaluation of the Individual Errors

All translative errors obtained as explained under "Selection of Measurements" and as recapitulated in detail below will be evaluated according to Rules 4 and 5 given in Sec. 4.5.

Construction of Translative Errors at Lines Other Than A, B, and C

The construction (computation) of the individual translative errors as stated above will be carried out as follows.

Lateral Errors.

Motion X. $\delta_z(x)$ (Fig. 38)—For primary effects, offsets b_2 and b_3 will be applied in combination with roll $\epsilon_x(x)$ in the way explained in Fig. 15 so as to obtain $\delta_z(x, A_2)$, $\delta_z(x, A_3)$. Both of them should be modified for the secondary effect of $\epsilon_y(z)$. However, straightness of $\delta_z(x)$ has very little meaning for boring operations which would be the only ones needing the whole motion z_{max} . Facing will be done at different depths with minimum necessary spindle extensions and the saddle always closest to the column. Therefore, the motion Z does not apply here nor does the secondary effect of $\epsilon_y(z)$. Therefore, errors $\delta_z(x, A_2)$, (δ_z, A_3) will be evaluated directly against the Tolerance Rule 4.

$\delta_y(x)$ (Fig. 39)—For primary effects, offsets c_4 and c_5 will be applied in combination with roll $\epsilon_x(x)$ so as to obtain $\delta_y(x, A_4)$, $\delta_y(x, A_5)$ along the extremes of the Working Space. Both of them will have to be modified

for the secondary effect of $\epsilon_z(z)$. This being the exceptional case of combination of primary and secondary effects as described in Sec. 4.3, it will have to follow Eq. (11).

Motion Y. $\epsilon_x(y)$ (Fig. 40)--For the primary effects, offsets c_2 and c_3 will be applied in combination with roll $\epsilon_y(y)$ so as to obtain $\delta_x(y, B_2)$ and $\delta_x(y, B_3)$ at the extreme of the Working Space. Both of these will have to be modified for secondary effects by expanding their error fields by tilting each by the corresponding differences $\epsilon_z(x, B_2) - \epsilon_z(x)_{\min} + \epsilon_z(z, B_2) - \epsilon_z(z)_{\min}$, $\epsilon_z(x, B_2) - \epsilon_z(x)_{\max} + \epsilon_z(x, B_2) - \epsilon_z(z)_{\max}$ and $\epsilon_z(x, B_3) - \epsilon_z(x)_{\min} + \epsilon_z(z, B_3) - \epsilon_z(z)_{\min}$, $\epsilon_z(z, B_3) - \epsilon_z(z)_{\max}$ according to Sec. 4.2, Table 2 (c) and (d), and Sec. 4.3, Eq. (10).

$\delta_x(y)$ (Fig. 41)--There is no primary effect because there is no applicable offset. The measurement $\delta_x(y, B)$ should, however, be modified for the secondary effects of the roll $\epsilon_x(x)$ and of the pitch $\epsilon_x(z)$. For the same reasons as in $\delta_x(x)$ above the latter effect will in practice not occur. Thus, the modification will only include tilts by differences $\epsilon_x(x, B) - \epsilon_x(x)_{\min}$ and $\epsilon_x(x, B) - \epsilon_x(x)_{\max}$. For illustration, see also Fig. 16 and Sec. 4.3, Eq. (10).

Motion Z. $\delta_x(z)$ (Fig. 42)--This is an error which will become effective in deep boring operations only and, therefore, is measured with maximum spindle extension. For the primary effect, offsets b_2 and b_3 apply together with roll $\epsilon_z(z)$ so as to obtain $\delta_x(z, C_2)$ and $\delta_x(z, C_3)$ at the extremes of the Working Space. Both of them will be modified for the secondary effect of the yaw $\epsilon_y(x)$, by applying to the error of direction e_d the differences $\epsilon_y(x, C) - \epsilon_y(x)_{\min}$, $\epsilon_y(x, C) - \epsilon_y(x)_{\max}$.

$\delta_y(z)$ (Fig. 43)--Because this error affects boring only, which with the Z motion active over the whole range is done with maximum spindle extension only, there is no primary effect. The measurement $\delta(z, C)$ must be modified for the secondary effect of the roll $\epsilon_x(x)$ by applying the differences $\epsilon_x(x, C) - \epsilon_x(x)_{\min}$ and $\epsilon_x(x, C) - \epsilon_x(x)_{\max}$.

Positioning Errors. There are no secondary effects on positioning errors.

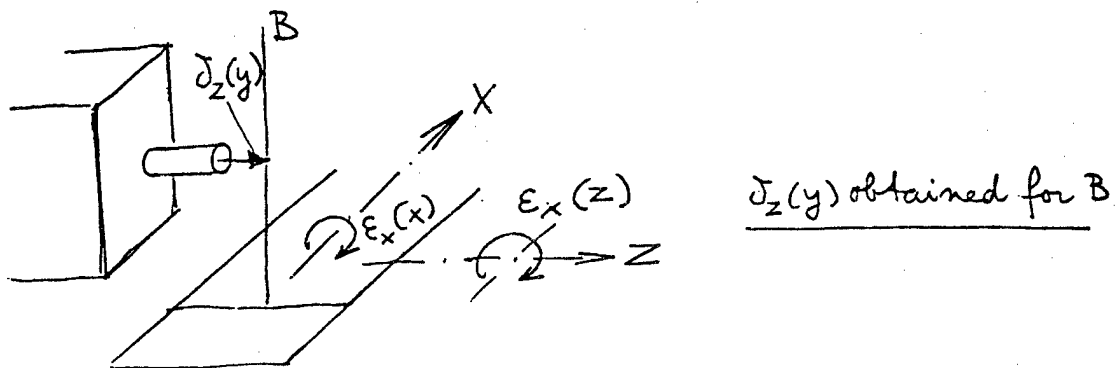


FIG. 41. Measurement of $\delta_z(y)$ (Y axis motion).

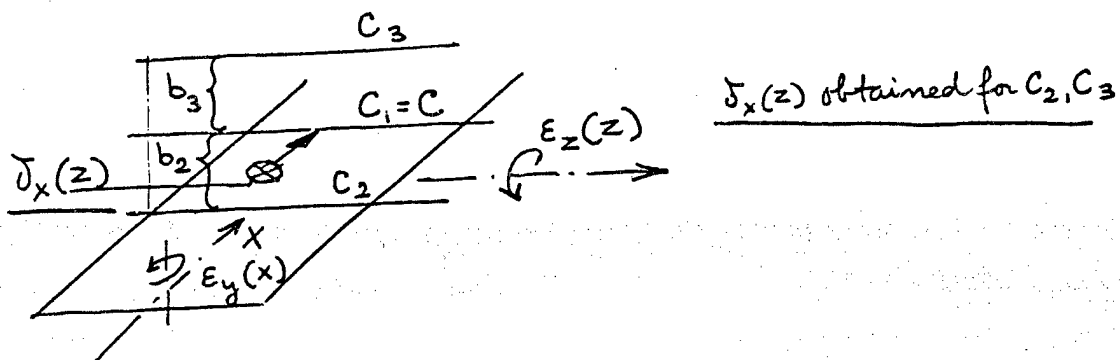


FIG. 42. Measurement of $\delta_x(z)$ (Z axis motion).

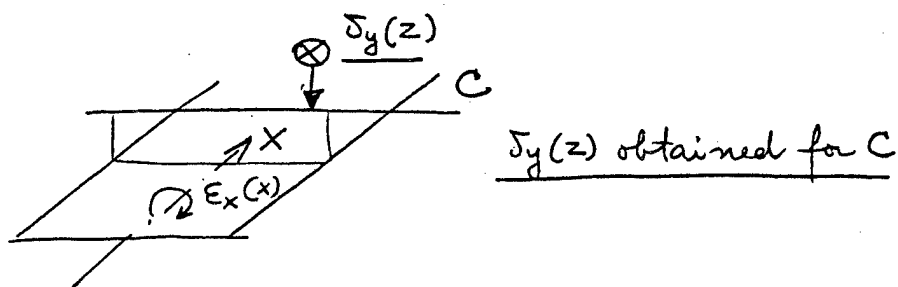


FIG. 43. Measurement of $\delta_y(z)$ (Z axis motion).

Motion X. $\delta_x(x)$ (Fig. 44)--For primary effects, offsets b_2 and b_3 apply in combination with pitch $\epsilon_z(x)$ and offsets c_2 and c_3 in combination with yaw $\epsilon_y(x)$. The errors along lines A_6 , A_7 , A_8 , and A_9 along the extremes of the Working Space will first be obtained by operations:

$$\begin{aligned}\delta_x(x, A_6) &= \delta_x(x, A_1) - b_2 \epsilon_z(x) - c_4 \epsilon_y(x) \\ \delta_x(x, A_7) &= \delta_x(x, A_1) - b_3 \epsilon_z(x) - c_4 \epsilon_y(x) \\ \delta_x(x, A_8) &= \delta_x(x, A_1) - b_2 \epsilon_z(x) - c_5 \epsilon_y(x) \\ \delta_x(x, A_9) &= \delta_x(x, A_1) - b_3 \epsilon_z(x) - c_5 \epsilon_y(x)\end{aligned}$$

The so-obtained errors $\delta_x(x, A_6)$, $\delta_x(x, A_7)$, $\delta_x(x, A_8)$, and $\delta_x(x, A_9)$ must be compared individually with the Tolerance Rule 4.

Motion Y. $\delta_y(y)$ (Fig. 45)--For the primary effect, offsets c_2 and c_3 apply in combination with yaw $\epsilon_x(y)$. The errors $\delta_y(y, B_2)$ and $\delta_y(y, B_3)$ must be individually evaluated against the Tolerance Rule 4.

Motion Z. $\delta_z(z)$ (Fig. 46)--For the primary effect, the offsets b_2 and b_3 apply in combination with the pitch $\epsilon_x(z)$. The errors $\delta_z(z, C_3)$ and $\delta_z(z, C_3)$ must be individually evaluated against the Tolerance Rule 4.

Weight Effects

It can be estimated that all the measurements associated with motions X and Z will be affected by the weight of the workpiece, but not those linked to the Y motion. However, the out-of-line measurements made during the Y motion are subjected to secondary effects of the angular motions associated with X and Z. This means that apart from having done all the 17 measurements with the table not loaded by a workpiece the following measurements have to be done again, with the table loaded with the maximum workpiece. Maximum is understood as such a weight and its distribution over the table for which the customer still requires guaranteed accuracy. For this, however, different A and K parameters of the tolerance template (Fig. 10) may be chosen. The size of the load must also be so chosen as not to prevent the location of the measuring instruments on the table.

The measurements to repeat are:

For motion X: $\delta_x(x)$, $\delta_y(x)$, $\delta_z(x)$ along line A and $\epsilon_x(x)$,
 $\epsilon_y(x)$, $\epsilon_z(x)$.

For motion Y: $\delta_x(y)$, $\delta_z(y)$, along line B.

For motion Z: $\delta_x(z)$, $\delta_y(z)$, $\delta_z(z)$ along line C and $\epsilon_x(z)$,
 $\epsilon_y(z)$, $\epsilon_z(z)$.

The above translative errors have to be constructed along the extreme offsets in the way given in the preceding "Construction of Translative Errors."

The so-constructed translative errors have to be subjected separately to corresponding tolerance templates according to Rule 4.

Thermal Effects

The environmental effects test will be done according to Sec. 7.1. The effect of internal sources will be done according to Sec. 7.2. We assume that only measurements associated with the motion Y are affected. A certain spindle speed will be determined and agreed between the supplier and the buyer for which in continuous run the accuracy specifications of the machine should still be met. Measurements will be made repeatedly during such a run for a specified period of time, and also after that at standstill for another specified period of time. The simplified way of measuring the drifts for the ends of the error graphs will be accepted as depicted in Fig. 35.

These drifts will be added to the ends of the lines enclosing the error fields obtained in the tests carried out initially with no heat sources on as they were described in the preceding "Construction of Translative Errors" (this applies to the third, fourth, and eighth cases described there).

For illustration, an example will be given here (see Fig. 47). Lines 1 and 2 enclose the $\delta_x(y)$ error as obtained from the $\delta_x(y)$ measurement along line B, and corrected for roll $\epsilon_y(y)$ so as to give $\delta_x(y, B_2)$. After the inclusion of the secondary effects of $\epsilon_z(x)$ and $\epsilon_z(z)$ this error field expands to a field enclosed by lines 3 and 4. The ends of lines 3 and 4 will move by drifts measured by gages 6 and 1 (shown on Fig. 35) to lines 5 and 6. In this way the total error field is now bounded by lines 5 and 6 and this whole field has to fit into the tolerance template according to Rule 4.

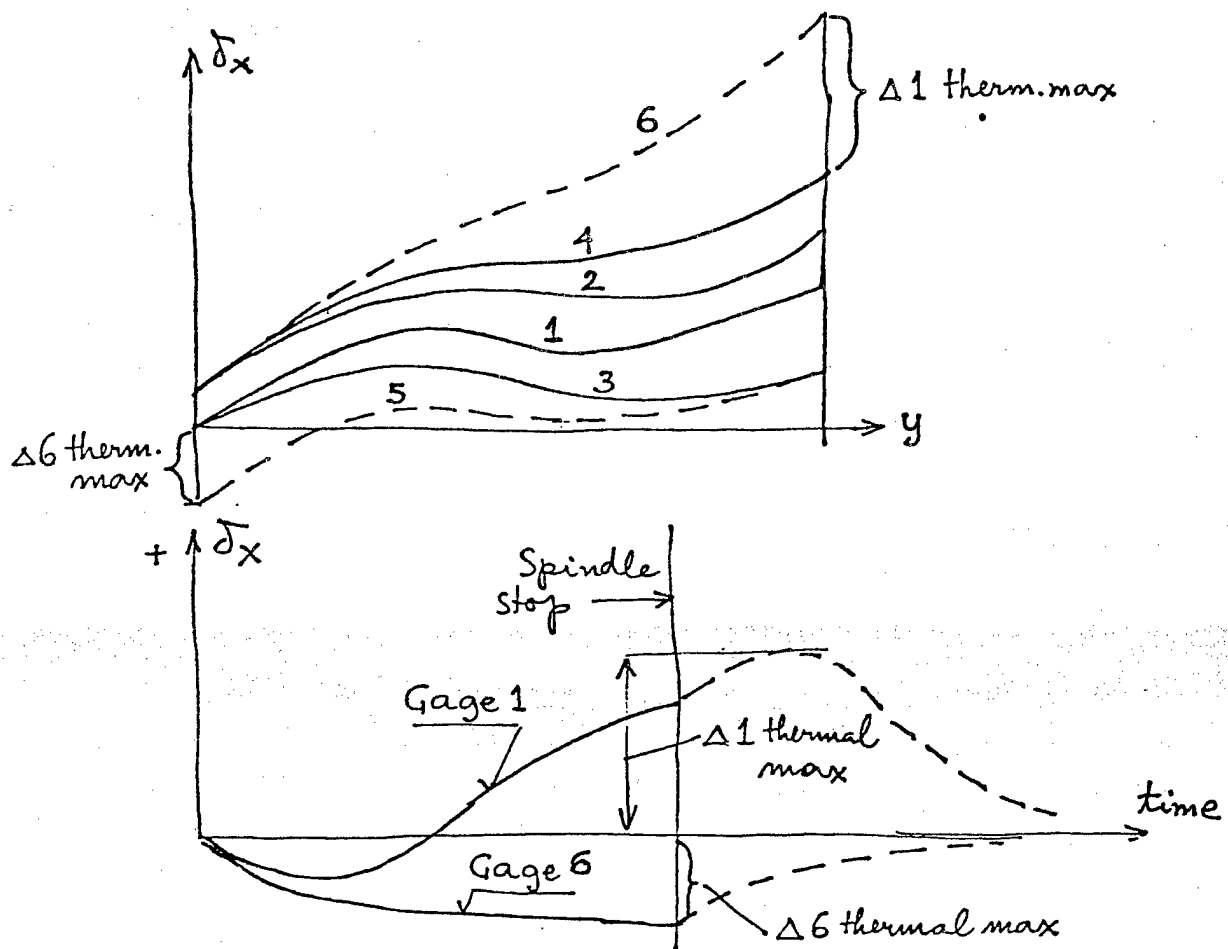


FIG. 47. Simulation of thermal effects errors as they apply to time and position.

9.0 SUMMARY OF ERRORS MEASURED IN INDUSTRY; TOLERANCE CLASSES

Here we present a survey of errors measured on 14 NC machine tools in various companies. These machines are mostly large. Some are newly installed and some have been used for a longer time. The fourth machine in Group A (see below) has been in use for 20 years. Table 3 lists the types of machine tools measured and the length of travel of the longest coordinate.

The various machines had different types of guideways--plain, rollertype, and hydrostatic--and different types of positional feedback transducers--synchros or encoders on leadscrews, Inductosyns, Ferranti linear gratings, measuring racks and pinions, and synchros. We will not try to associate these various design features with the individual errors. Instead, we will give a general summary of their accuracies.

9.1 POSITIONING ERRORS

Records of 40 positioning errors measured along the various axes of the various machines are reproduced in Figs. 48-58, in a sequence of increasing travel lengths. The scales for travel are given in millimeters (mm) or meters

TABLE 3. Machines measured in the survey of accuracies.

Group	Machine type	No. of machines measured	Longest travel (meters)
A	Horizontal boring and milling, floor type	5	6, 20, 20, 12, 6
B	Machining centers	3	1, 0.5, 2.5
C	Vertical spindle milling machines	3	3, 3, 0.63
D	Lathes	1	8
E	Plano milling machines, gantry type	2	12, 12

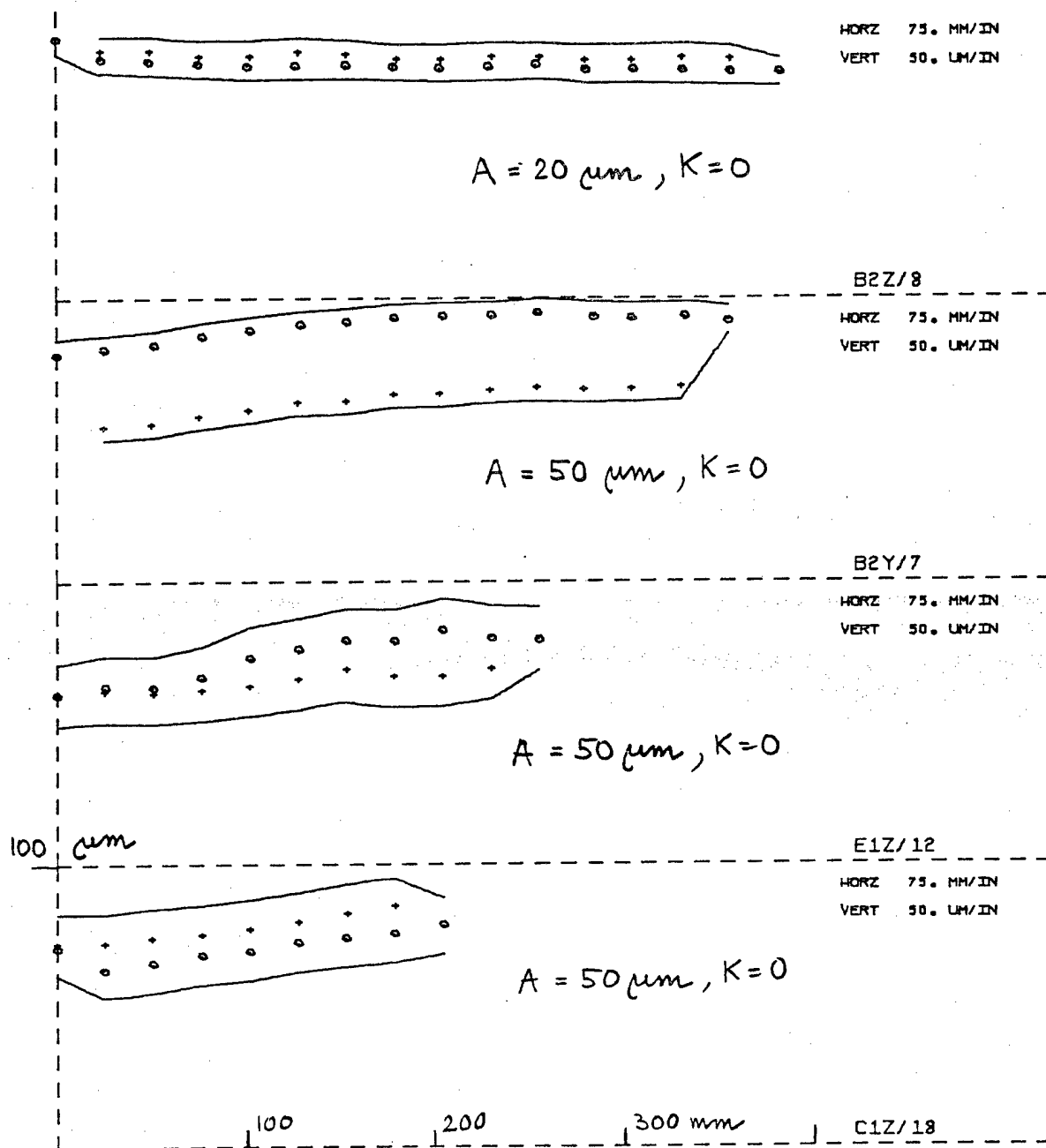


FIG. 48. Tolerance template parameters.

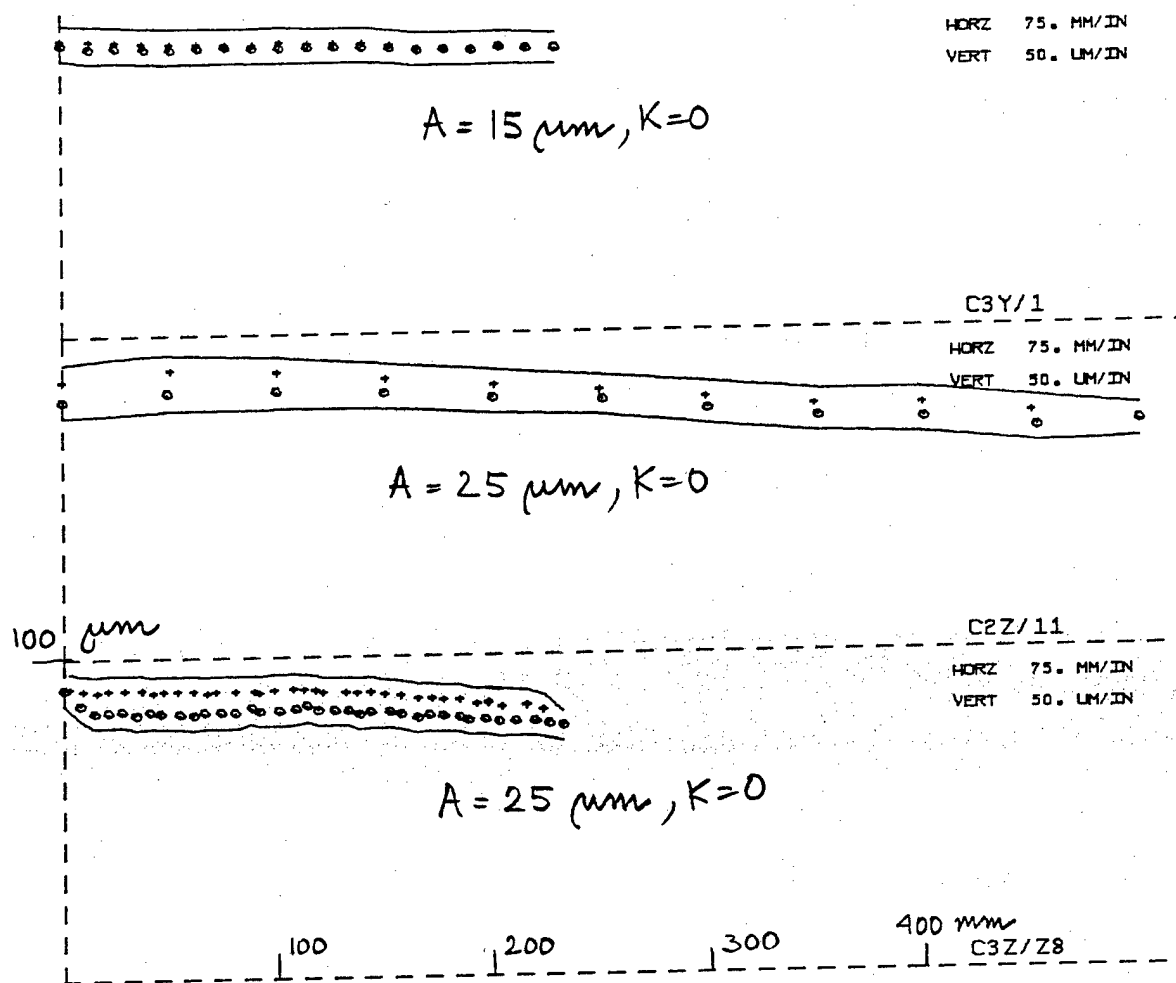


FIG. 49. Tolerance template parameters.

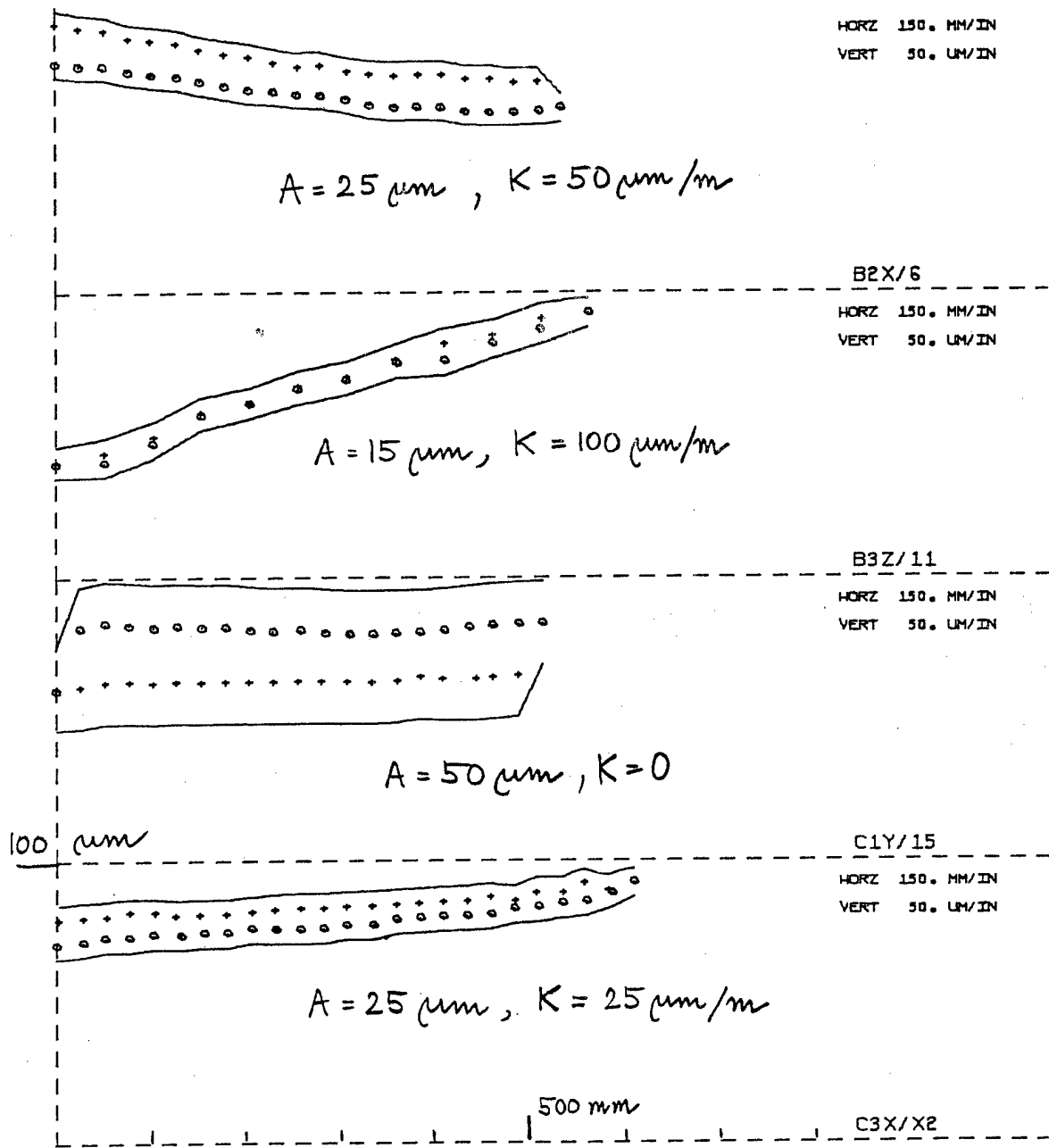


FIG. 50. Tolerance template parameters.

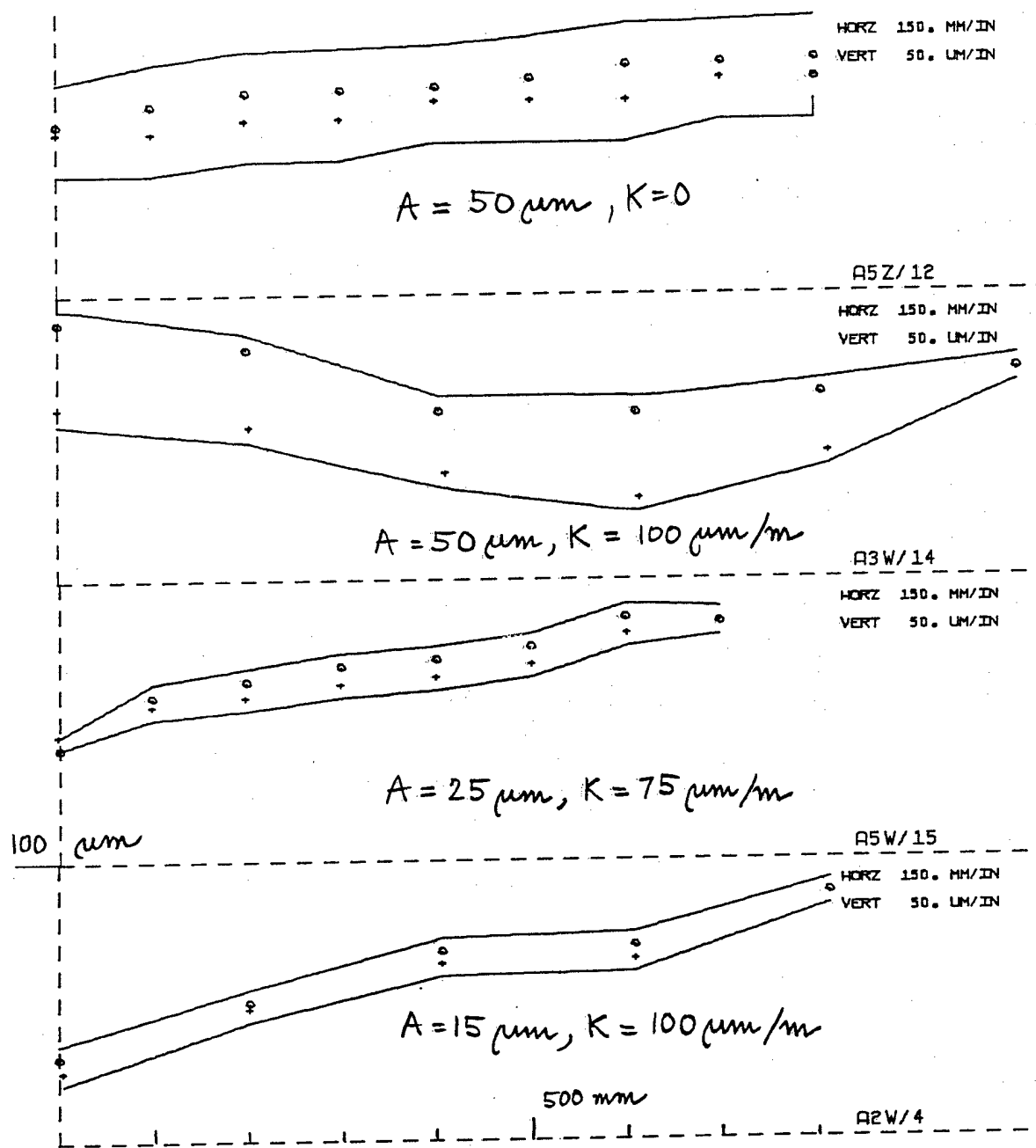


FIG. 51. Tolerance template parameters.

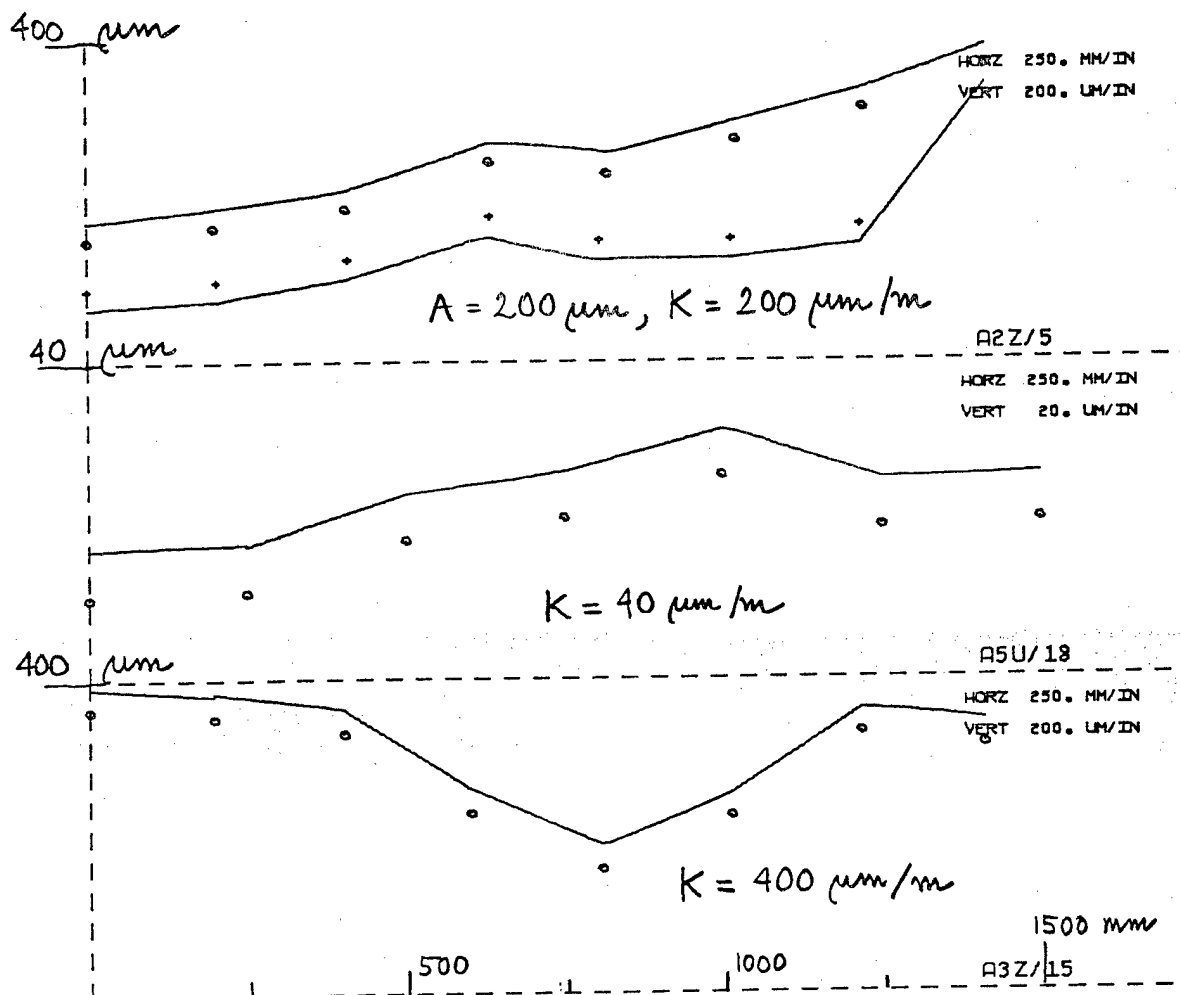


FIG. 52. Tolerance template parameters.

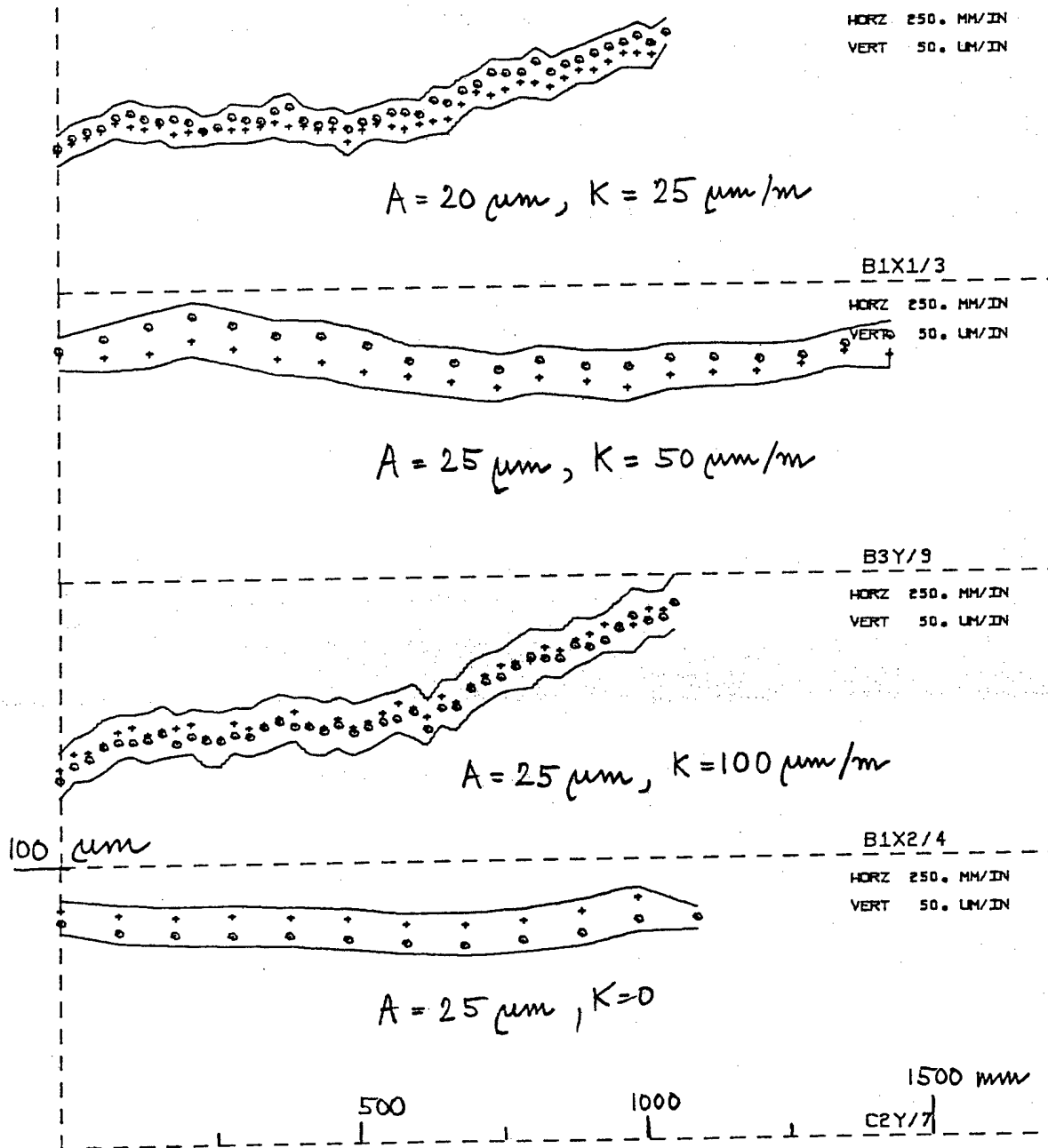


FIG. 53. Tolerance template parameters.

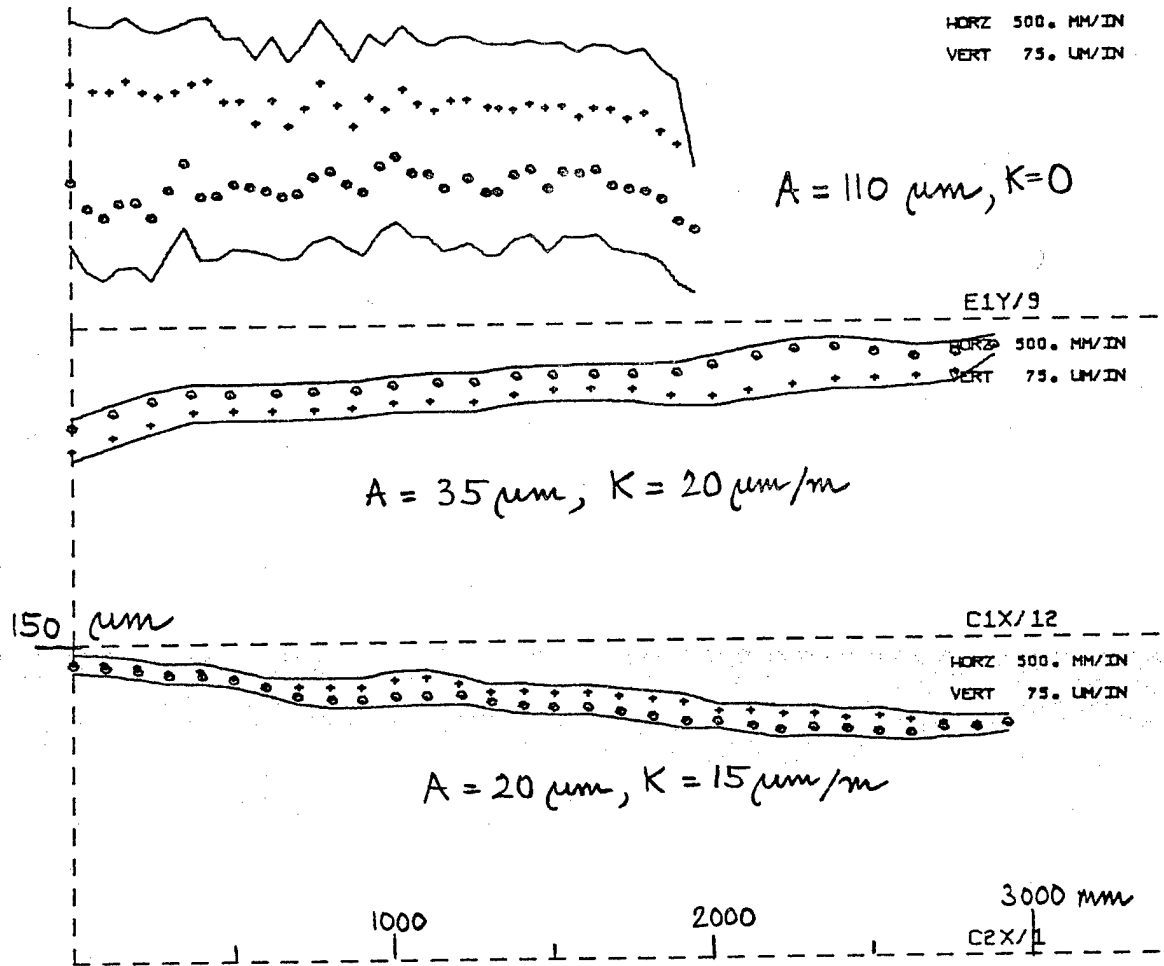


FIG. 54. Tolerance template parameters.

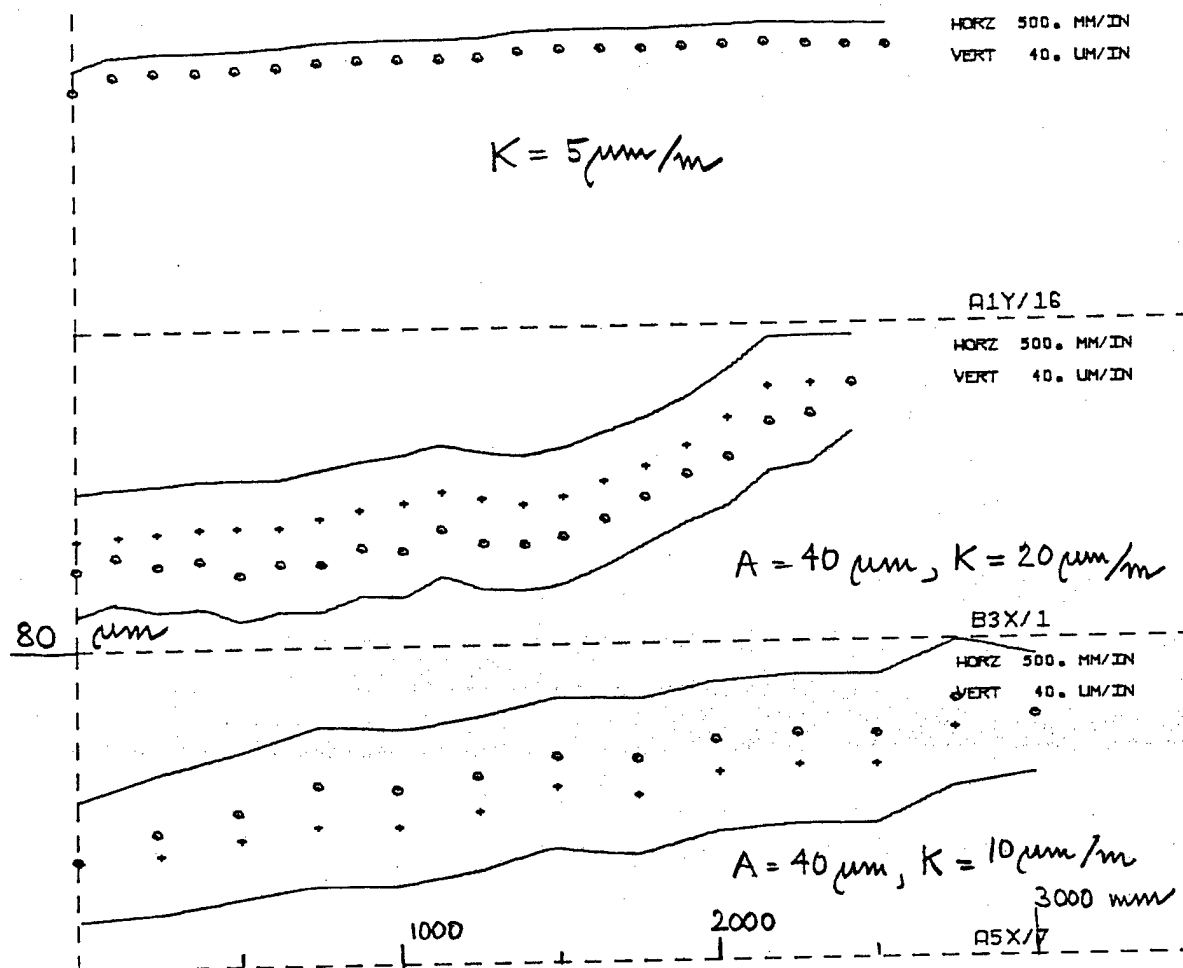


FIG. 55. Tolerance template parameters.

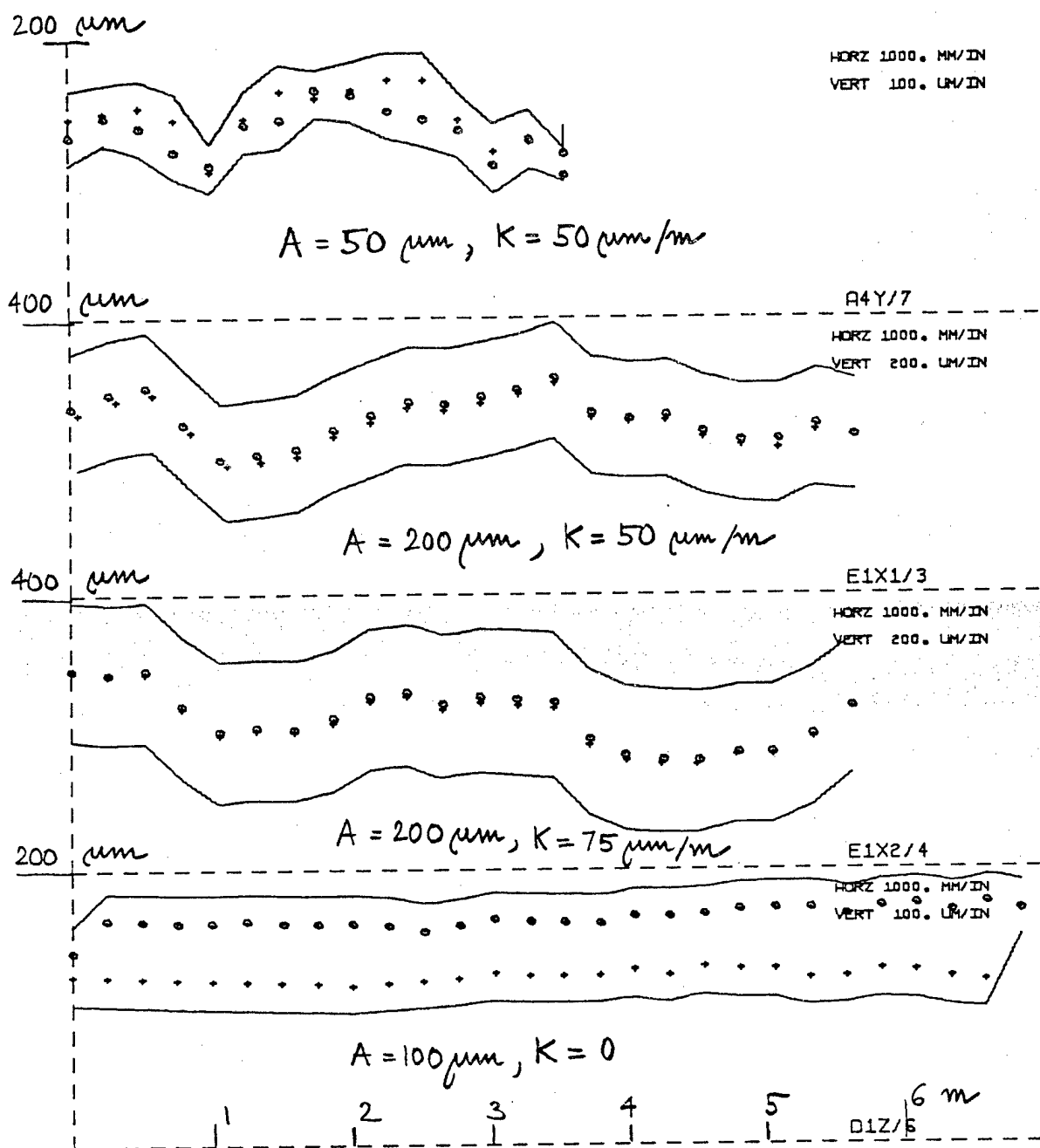


FIG. 56. Tolerance template parameters.

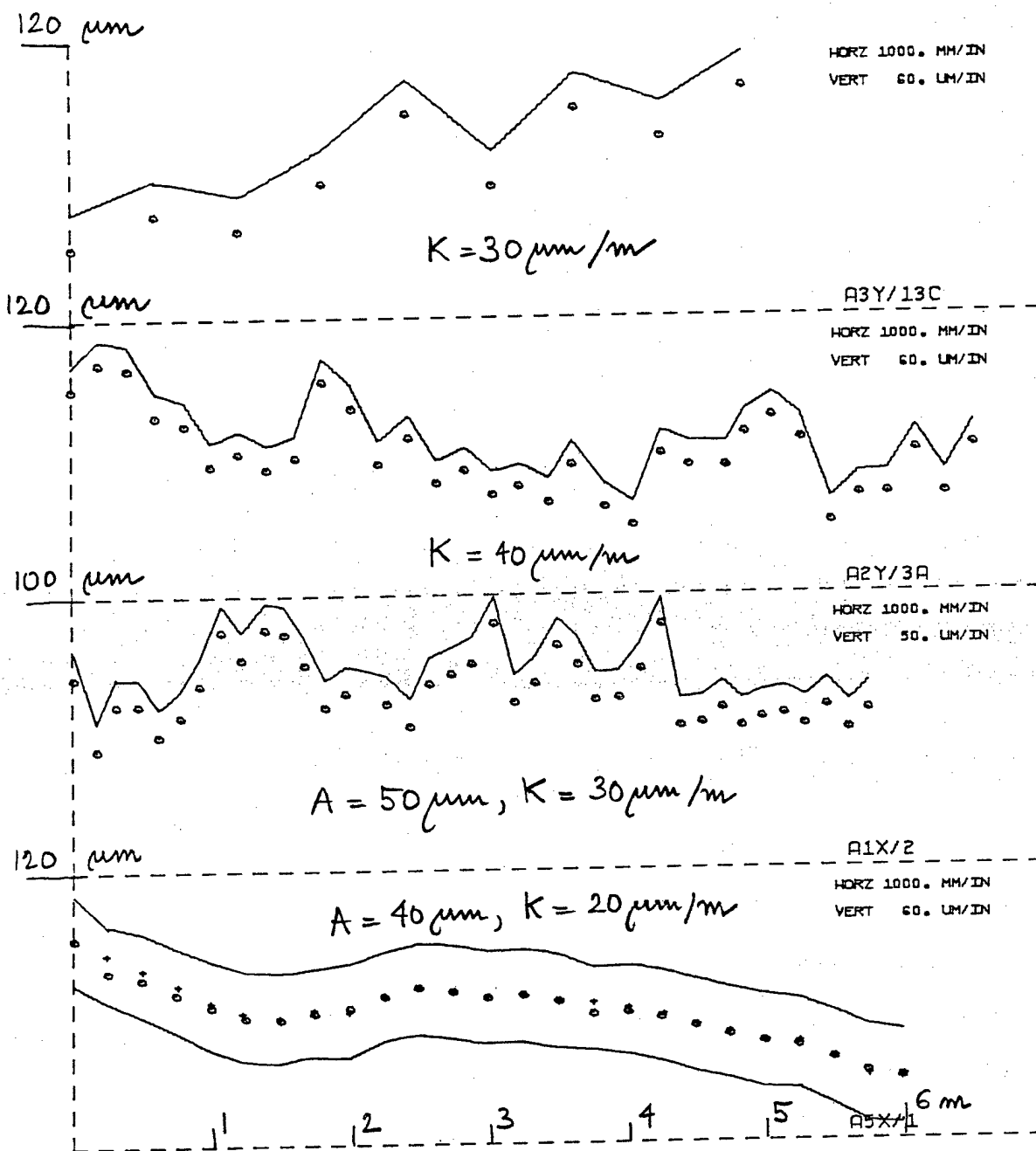


FIG. 57. Tolerance template parameters.

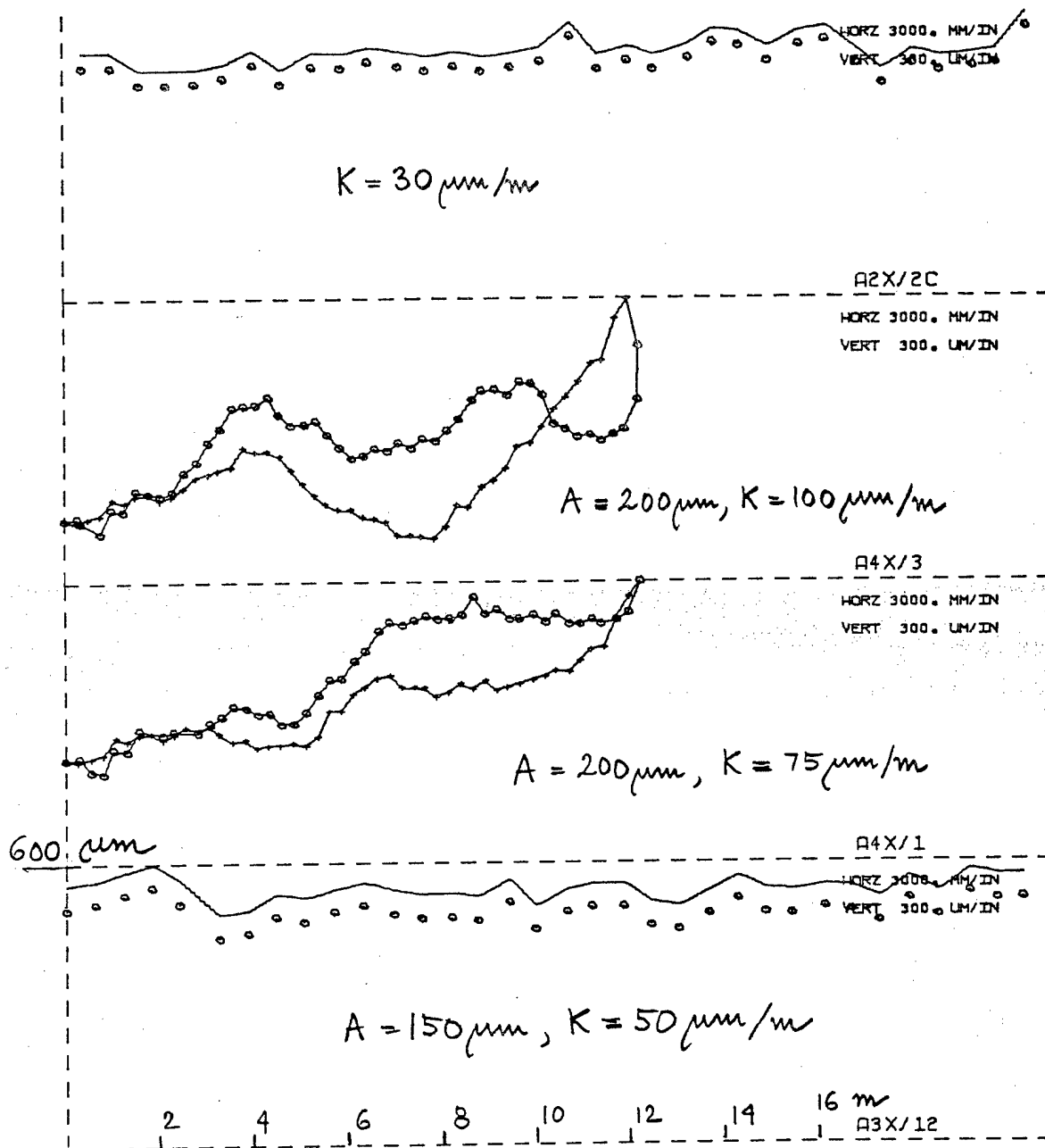


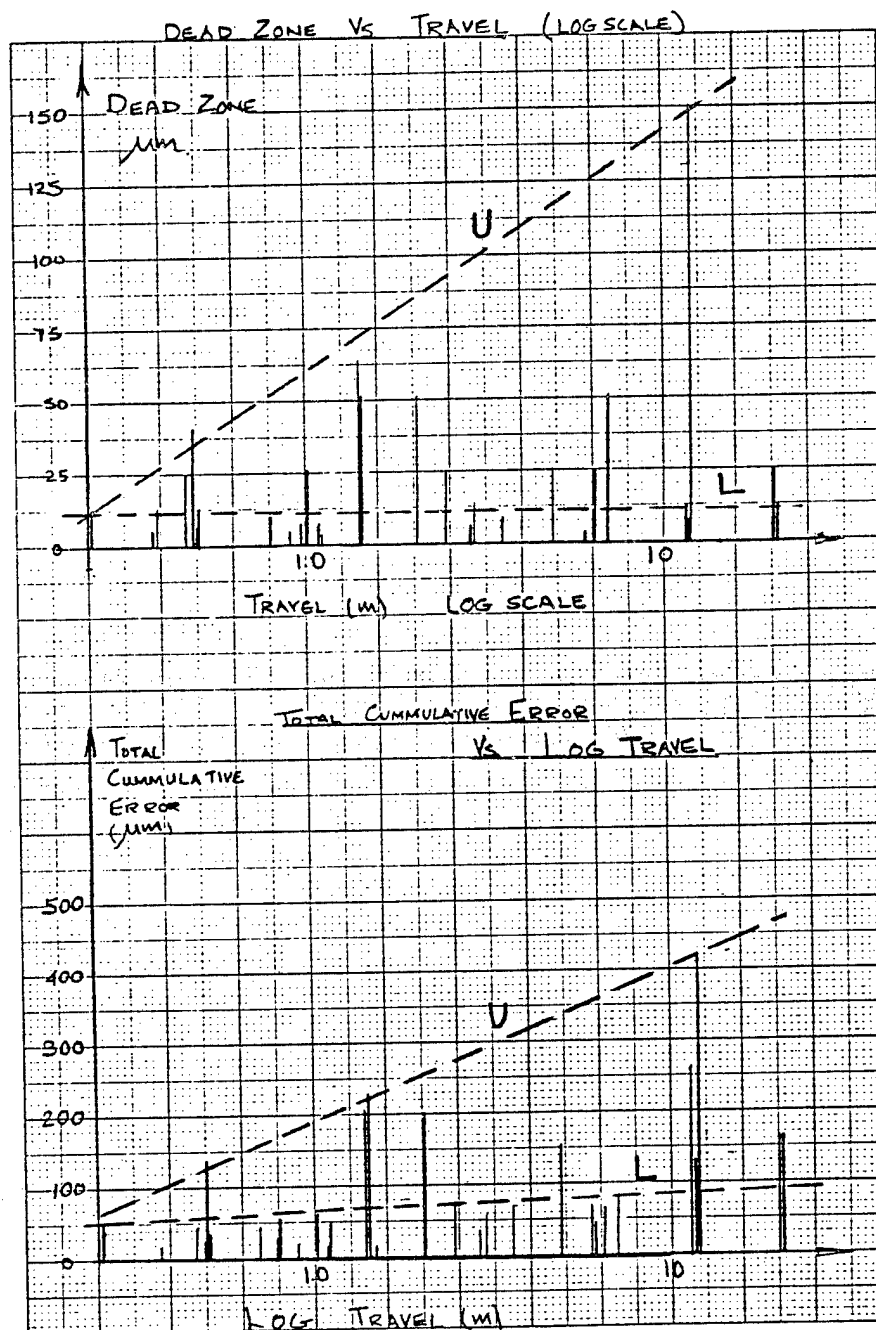
FIG. 58. Tolerance template parameters.

(m) and for errors in micrometers (μm). In most cases errors measured during travel in both + and - directions are indicated by circles and crosses, the difference between the two being the dead zone. The lines enclosing the error fields include scatter and short-period periodic errors (once per revolution of measuring pinion, once per revolution of leadscrew, etc.), as added on both sides of the circles and crosses representing the mean positions. In every graph the machine tool group (A, B, . . .) from Table 3, the number of the machine in the group (in the sequence in which the longest travels are given in Table 3), and the axis (e.g.: B2Z/8) are given. Here the number 8 is an internal identification number. Each graph is evaluated by the values of the parameters A and K of the tolerance template into which the graph would fit, according to the definition given in Sec. 3.6 by Eq. (3), as well as in Fig. 10. In some of the graphs $K = 0$. This applies mostly to the short travels, but also to some of the long travels where it signifies a very good adjustment of the feedback device over long distance. In those graphs where measurement was made in one direction only the parameter A is not given.

All of these graphs are summarized in Figs. 59-61. In Fig. 59 the dead zone and the "total cumulative positioning error" (maximum positioning error) are plotted versus travel length (in log scale). In more than half of the cases these errors almost do not depend on the travel length (broken line L) and they are below $10\ \mu\text{m}$ ($0.0004\ \text{in.}$) for the dead zone and below 50 to $100\ \mu\text{m}$ (0.002 to $0.004\ \text{in.}$) for the maximum error. In other cases these errors increase with the travel length; i.e., with the size of the machine (broken line U).

A similar situation occurs for the "scatter" as shown in Fig. 60, where the L line limits values to $2.5\ \mu\text{m}$ ($0.0001\ \text{in.}$).

The values of the A and K parameters of the tolerance template are given by points and circles in the graphs of Fig. 61. These values do not seem to depend much on the size of the machine. They could be grouped into three levels L1, L2, and L3 with the two lower levels L1 and L2 including approximately equal numbers of cases and the third level established for the "exceptions," which however tend to associate with the larger machines. The values of the parameters in these levels are:



POSITIONING

FIG. 59. Dead zone and total cumulative error vs travel (summarized data).

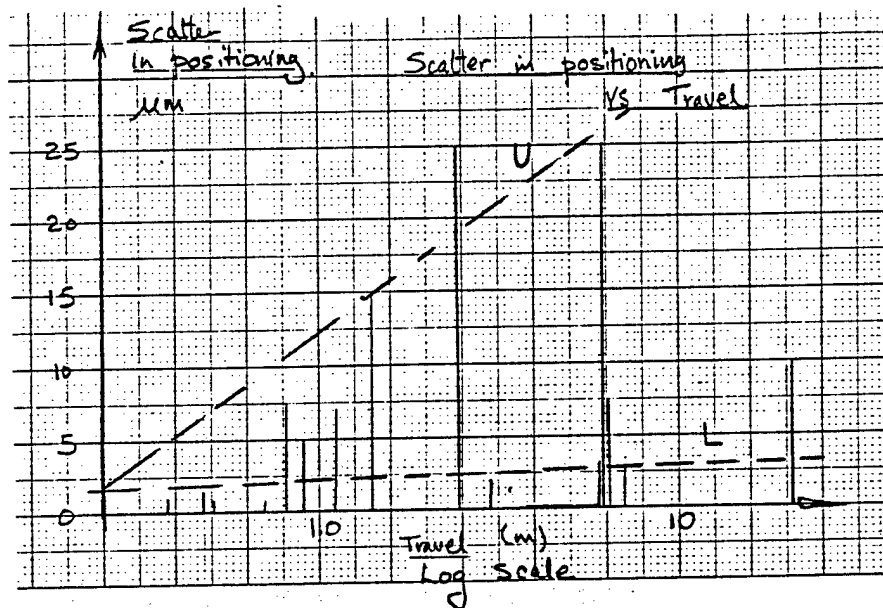


FIG. 60. Scatter vs travel (summarized data).

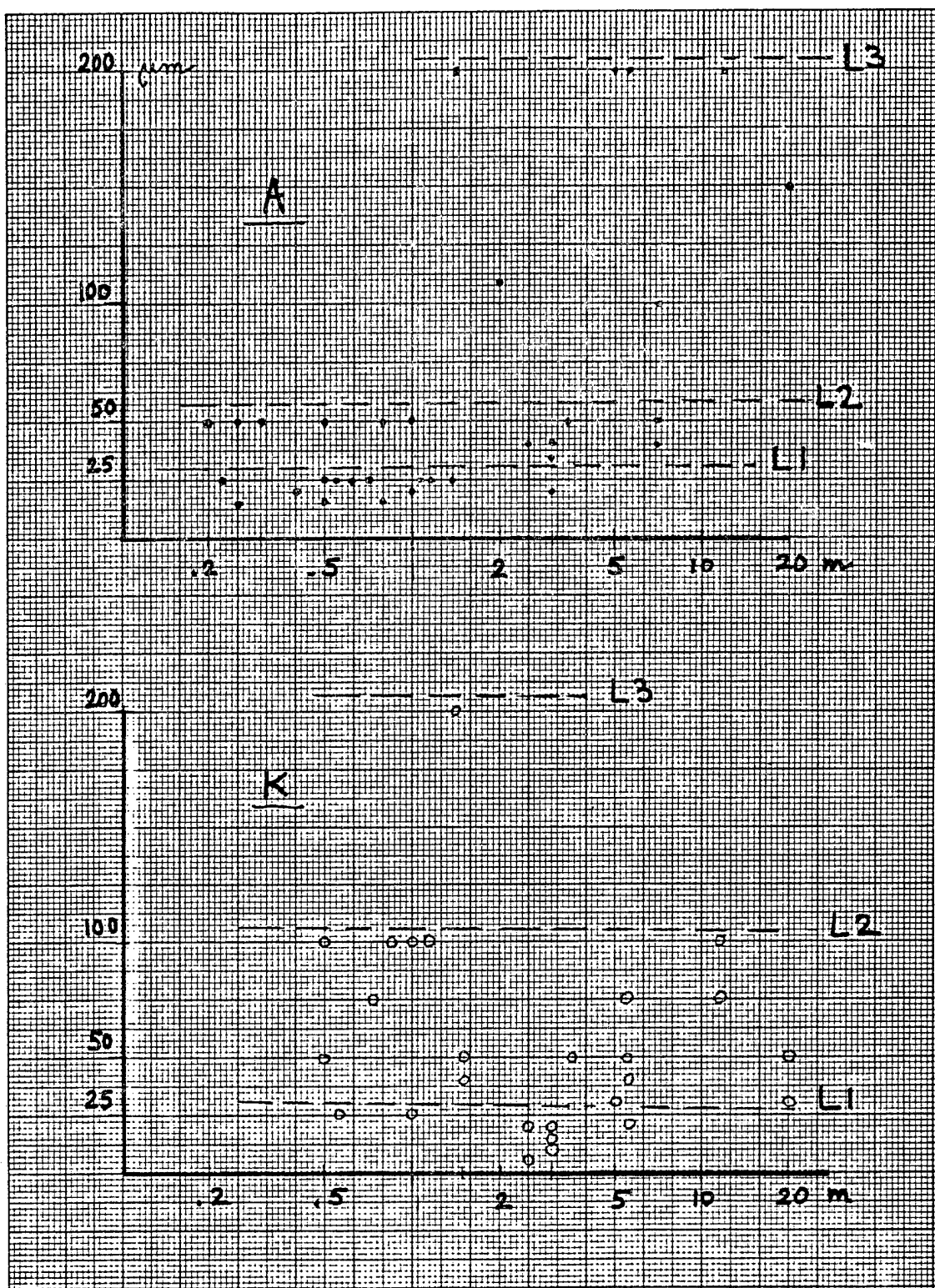


FIG. 61. Tolerance template parameters A and K vs size and levels L_1 , L_2 , L_3 .

L1, A = 25 μm (0.001 in.), K = 25 $\mu\text{m}/\text{m}$ (0.001 in./yard)
 L2, A = 50 μm (0.002 in.), K = 100 $\mu\text{m}/\text{m}$ (0.004 in./yard)
 L3, A = 200 μm (0.008 in.), K = 200 $\mu\text{m}/\text{m}$ (0.008 in./yard)

9.2 LATERAL ERRORS

We have much less data about the lateral errors. This is mainly due to the fact that the measurement of straightnesses of motions over larger travels is more tedious than positioning measurements. In the past we have mostly used rather short straightedges and squares and measured over only parts of the travels. Therefore, our data for lateral errors is less reliable than for positioning. It was not until recently that these measurements were done according to the rules recommended in the preceding parts of this report. Nevertheless, we consider it useful to present a global summary of the measured errors. These are given in Fig. 62.

The upper graph represents the "straightness error e_s " as defined in Sec. 3.1 and in Fig. 3. The lower graph represents the squareness and parallelity errors, i.e., the "errors of direction e_d ." The dead zones of the lateral errors are presented in Fig. 63.

All these errors in the three graphs are arbitrarily grouped into two levels:

L1, $e_s = 2.5 \mu\text{m}$ (0.0001 in.), $e_d = 25 \mu\text{m}/\text{m}$ (0.001 in./yd),
 DZ = $s 2.5 \mu\text{m}$ (0.0001 in.)
 L2, $e_s = 33 \mu\text{m}$ (0.0013 in.), $e_d = 125 \mu\text{m}/\text{m}$ (0.005 in./yd),
 DZ = $s 15 \mu\text{m}$ (0.0006 in.).

It may now be assessed that these values would lead to the following parameters of the tolerance templates:

L1, A = 12.5 μm (0.0005 in.), K = 25 $\mu\text{m}/\text{m}$ (0.001 in./yd)
 L2, A = 50 μm (0.002 in.), K = 125 $\mu\text{m}/\text{m}$ (0.005 in./yd).

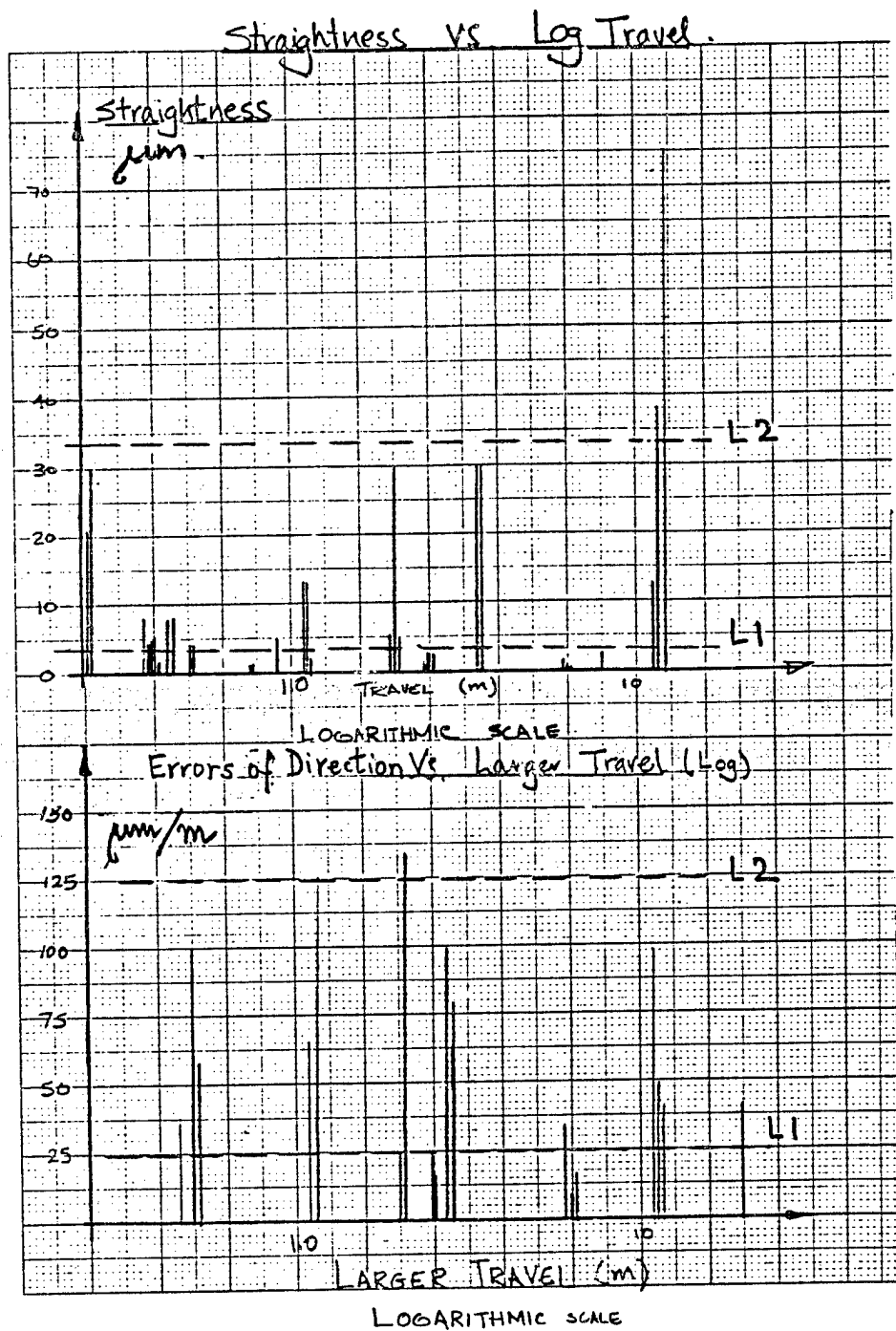


FIG. 62. Straightness and directional errors vs travel (summarized data).

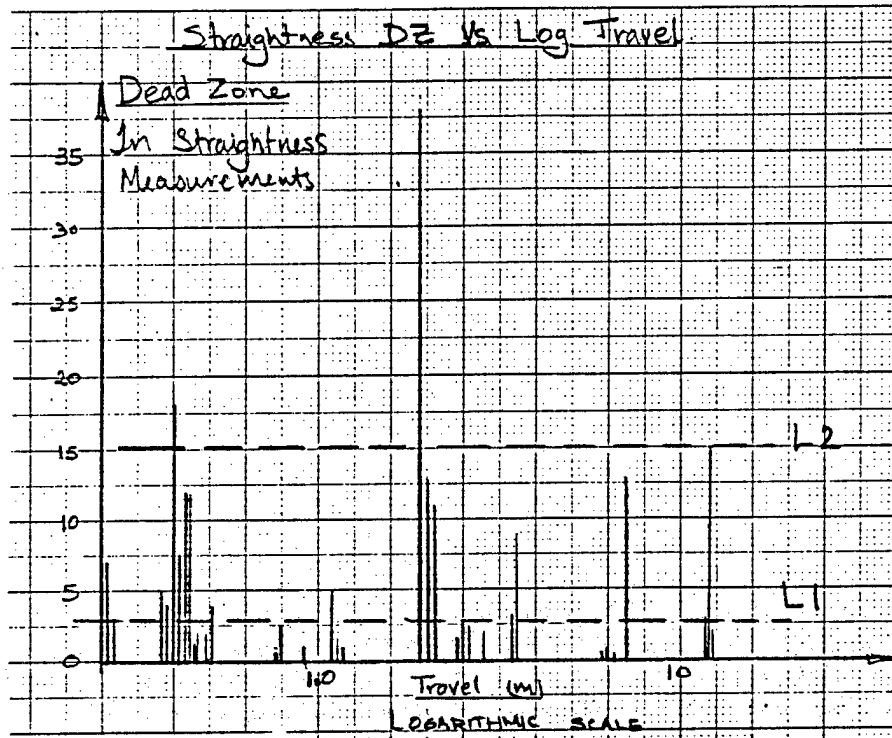


FIG. 63. Straightness dead zone vs travel (summarized data).

9.3 ANGULAR MOTION ERRORS

In most instances the measurements of the translative errors were made along selected lines in the most-used parts of the working space; not, however, at the locations of the feedback transducers, which is mostly the practice used by the machine tool suppliers. In this way, the data in the two preceding paragraphs about positioning the lateral errors is reasonably representative of the accuracies imprinted on the workpieces. Angular motion errors--pitch, yaw and roll--were not measured systematically in the past. It is necessary to say that in some instances rather large values of these errors were found which were due to insufficient alignment of the beds of the machines.

For the evaluation of errors over the whole working spaces of machine tools the effects of the angular motions will have to be taken into account.

9.4 TOLERANCE CLASSES

As a guideline, one might think about establishing "Accuracy Classes" of machine tools. If this should be done on the basis of accuracies encountered in practice, the values of the A and K parameters of the tolerance templates as given above might be considered for measurements along the centerlines of the working spaces with, perhaps, double those values for the entire working spaces. There would, of course, still have to be a "superclass LO" with tighter tolerances to be specified in the individual cases where such high accuracies will be required.

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APPENDIX DETERMINING EFFECTS OF ANGULAR MOTIONS

Primary effects affect all the three translative errors. Secondary effects affect the directional part of the lateral errors only.

PRIMARY EFFECTS

If the measurements $\delta_x(i)$, $\delta_y(i)$, $\delta_z(i)$, where $i = x, y, z$, were made along a line P_1 and then repeated along another line P_2 that is offset in the other coordinates than i with respect to P_1 , the results will be different due to angular errors $\epsilon_x(i)$, $\epsilon_y(i)$, $\epsilon_z(i)$ and offsets ΔX , ΔY , ΔZ , ΔZ^* (whichever exist), and the difference will be

$$\Delta \begin{Bmatrix} \delta_x(i) \\ \delta_y(i) \\ \delta_z(i) \end{Bmatrix} = \Delta \begin{bmatrix} 0 & Z+Z^* & Y \\ Z+Z^* & 0 & X \\ Y & X & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_x(i) \\ \epsilon_y(i) \\ \epsilon_z(i) \end{Bmatrix} \quad (A-1)$$

Equation (A-1) is explained in Fig. A-1, where it is shown that rotation ϵ_i around the axis i causes differences

$$\begin{aligned} \Delta \delta_k &= \epsilon_i \Delta J \\ \Delta \delta_j &= \epsilon_i \Delta K \end{aligned}$$

In the individual particular cases it is necessary to check which offsets do not exist and replace them by zeros in the square matrix of Eq. (A-1). In all instances $\Delta I = 0$.

In the particular case of the horizontal boring machine it is obvious that the tool can be offset with respect to the individual carriers as given in the following table.

TABLE A-1. Possible offsets of the horizontal boring machine.

Carrier	Motion	ΔX	ΔY	ΔZ	ΔZ^*
Table	X	0	1	1	1
Headstock	Y	0	0	0	1
Saddle	Z	0	1	0	1

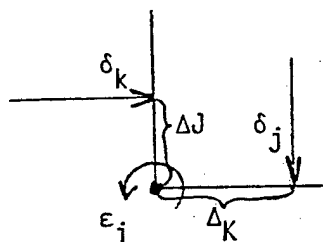


FIG. A-1.

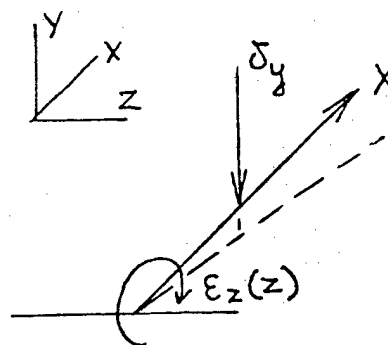


FIG. A-2.

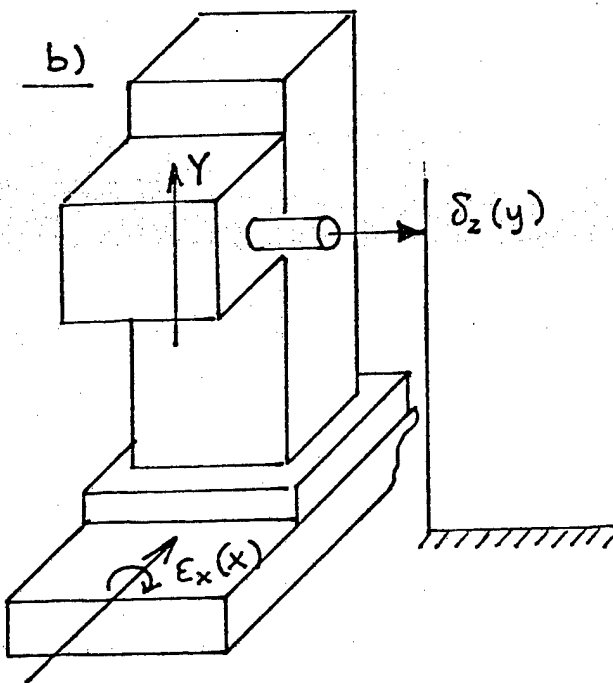
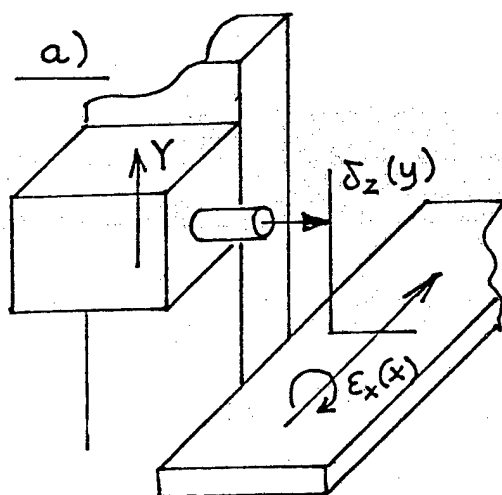


FIG. A-3.

Correspondingly, Eq. (A-1) applies to the individual motions of this machine as follows.

Motion X:

$$\Delta \begin{Bmatrix} \delta_x(x) \\ \delta_y(x) \\ \delta_z(x) \end{Bmatrix} = \Delta \begin{bmatrix} 0 & Z+ & Y \\ Z+ & 0 & 0 \\ Y & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_x(x) \\ \epsilon_y(x) \\ \epsilon_z(x) \end{Bmatrix} \quad (\text{A-2a})$$

Motion Y:

$$\Delta \begin{Bmatrix} \delta_x(y) \\ \delta_y(y) \\ \delta_z(y) \end{Bmatrix} = \Delta \begin{bmatrix} 0 & Z & 0 \\ Z & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_x(y) \\ \epsilon_y(y) \\ \epsilon_z(y) \end{Bmatrix} \quad (\text{A-2b})$$

Motion Z:

$$\Delta \begin{Bmatrix} \delta_x(z) \\ \delta_y(z) \\ \delta_z(z) \end{Bmatrix} = \Delta \begin{bmatrix} 0 & Z^* & Y \\ Z^* & 0 & 0 \\ Y & 0 & 0 \end{bmatrix} \begin{Bmatrix} \epsilon_x(z) \\ \epsilon_y(z) \\ \epsilon_z(z) \end{Bmatrix} \quad (\text{A-2c})$$

The effects expressed by Eq. (A-1) are maximum at maximum values of offsets.

SECONDARY EFFECTS

Secondary effects affect angularly the relative position of the Master Part and the tool. Therefore, they cause changes in the individual deviations which are proportional to the distance traveled (see Fig. A-2).

$$\Delta \delta_y(x) = \epsilon_z(z) \cdot X$$

Furthermore, it is obvious that only those angular errors that either rotate the workpiece or carrier of a tool motion affect a particular lateral error.

As an illustration let us look at the $\delta_z(y)$ error as measured (a) on a table-type and (b) a floor-type horizontal boring machine (see Fig. A-3). In the former case it is the roll $\epsilon_x(x)$ of the table carrying the workpiece which affects $\delta_z(y)$ and in the latter case it is the roll of the column which carries the headstock which is the carrier of the Z coordinate motion.

If the effect exists, then it is in general of the form:

$$\Delta \begin{Bmatrix} \delta_j(i) \\ \delta_k(i) \end{Bmatrix} = i \Delta \begin{Bmatrix} \epsilon_k(j) + \epsilon_k(k) \\ \epsilon_j(j) + \epsilon_j(k) \end{Bmatrix} \quad (A-3)$$

where the vector on the right hand side represents differences Δ of the angle ϵ with respect to those obtained at values of j and k at which the original measurements $\delta_j(i)$, $\delta_k(i)$ were carried out.

For the example of the horizontal boring machine, angular errors associated with motions X and Z rotate the workpiece and, therefore, apply. Angular errors associated with the Y motion do not apply because motion Y does not carry a tool carrier but the tool directly.

Equation (A-2) is then modified for this particular example as follows.

Motion X:

$$\Delta \begin{Bmatrix} \delta_y(x) \\ \delta_z(x) \end{Bmatrix} = x \Delta \begin{Bmatrix} \epsilon_z(z) \\ \epsilon_y(z) \end{Bmatrix} \quad (A-4a)$$

Motion Y:

$$\Delta \begin{Bmatrix} \delta_x(y) \\ \delta_z(y) \end{Bmatrix} = y \Delta \begin{Bmatrix} \epsilon_z(x) + \epsilon_z(z) \\ \epsilon_x(x) + \epsilon_x(z) \end{Bmatrix} \quad (A-4b)$$

Motion Z:

$$\Delta \begin{Bmatrix} \delta_x(z) \\ \delta_y(z) \end{Bmatrix} = z \Delta \begin{Bmatrix} \epsilon_y(x) \\ \epsilon_x(x) \end{Bmatrix} \quad (A-4c)$$

The effects given on the left-hand sides of Eq. (A-3) are maximum for maximum values of the vectors on the right-hand side.

COMBINING PRIMARY AND SECONDARY EFFECTS

For measurements associated with a motion I the primary effects depend on the values of the other coordinates J and K, while the secondary effects depend on angular errors $\epsilon(j)$ and $\epsilon(k)$ associated with these other coordinates.

An easy combination is only then possible when a particular error $\delta(i)$ is primarily affected by the value of the one coordinate j and by the angular error of another coordinate $\epsilon(k)$. Only then can the maxima of the effects be reached independently.

Taking the examples of Eqs. (A-2) and (A-4) this condition is satisfied for:

$$\delta_x(x), \delta_z(x), \delta_x(y), \delta_y(y), \delta_z(y), \delta_x(z), \delta_y(z), \delta_z(z).$$

It is not satisfied for $\delta_y(x)$.